Problem Set: Fib-Newton

1. Find F_{15} and F_{16}, the 15th and 16th terms of the usual *Fibonacci Sequence*. Calculate the ratio: F_{16}/F_{15} and compare it with the number $\left(\sqrt{5}+1\right)/2$ (use as many decimal places as you can get from your calculator).

2. The basic rule for Fibonacci sequences is that any term is the sum of the two previous terms. The usual sequence starts with the first two terms 1, 1. Suppose we start the Fibonacci sequence with 3 and 6 instead of 1 and 1. Calculate the 15th and 16th terms and find their ratio as in question 1. Compare your new ratio with that from question 1. Try another sequence beginning with any two terms of *your* choosing and repeat these calculations. Any conjectures about the ratio F_{31}/F_{30}? Calculate it.

3. Suppose we start a Fibonacci sequence with the terms A and B; what will be the next term? What will be the fourth and fifth terms? Find a few more; in other words, write the Fibonacci sequence that begins: $A, B, \ldots$. You should simplify what you get by collecting all the A's together and all the B's together. Do you see a pattern in the coefficients of A and B?

   Suppose A and B are both *even*, what can you say about the "evenness" of the rest of the sequence (explain)? What happens if A and B are both *odd*? What if one is odd and the other is even? Work out a few examples for different values of A and B to see what's happening. Try to explain the patterns you see.

4. *Newton's Law of Gravity* says that the force between two given masses, d units apart, is given by:

$$F = \frac{K}{d^2}$$

   (where K is a constant depending on the masses and the units chosen). Suppose that the force F equals 8 *Newtons* (a unit of force) when the masses are 2.1 meters apart. Calculate the value of K (you'll probably need a calculator). Rewrite the formula, putting in the value of K you've just determined. Suppose that we double the distance between the masses, to 4.2 meters. What will be the new force? How is it related to the old force in size? Try using algebra to see what happens, in general, to the force when the distance is doubled, from d to $2d$. What will happen to the force when the distance goes from d to $10d$? (Try, for example, 21 meters instead of the original 2.1.).

5. Set up perpendicular axes on a piece of graph paper, with the vertical axis labelled "F"; mark it off in units of 50, from 0 to 650. Label the horizontal axis "d" and mark it off in units of $\frac{1}{4}$, from 0 to 4. Plot the relation between force F and distance d from the previous problem, plotting points at horizontal intervals of $\frac{1}{4} = 0.25$. Connect the dots with a smooth curve (note that you don't get a value for $d = 0$!).

(Note: If you know how to set up a spreadsheet, you might want to use it to do the problems above, except for #3. Writing a computer program is another good tool for investigating Fibonacci series. Please see me for suggestions.)