Syllabus for MTH U201: History of Mathematics

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Hours: M,W,Th, 1:00 - 1:30, and by appointment
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Text: *The History of Mathematics*, David M. Burton (6th ed, McGraw-Hill, 2007)

Course Requirements

1. Students are expected to keep up with the reading and all written questions relating to the reading. There may be *quizzes* on this material.
2. There will be *two required papers* on topics related to the course, counting a total of **30%** of the course grade. A list of suggested topics will be distributed. Each paper must be at least 5 pages *plus* a complete bibliography, including explicit web sites and sources of quotes.
3. There will be *several required assignments*—to be announced—counting, with quizzes, **22%**.
4. There will be a *midterm*, counting for **16%** of your grade, and a *final* counting **32%**.
5. Attendance will be taken, and anyone with *more than 3 cuts may be dropped* from the course. Late assignments will lose credit, and no assignment will be accepted more that 1 week late.
6. *Math majors* taking this course for math major credit *will be expected to do extra work* of a more technical nature, some of which is described in the assignments below. They should see me early in the quarter.
7. Please come to my office hours! This is not a technical course, but there are some topics where mathematics must be done. I will be glad to give you help if you need it. I also would be very pleased to just talk about the course, mathematics in general, or anything you'd like to discuss. I will post my office hours at the beginning of the quarter.
8. I usually check my e-mail several times a day; write if you want – I'll answer as soon as I can.

I. Arithmetic in the Ancient World

Counting and Egyptian numbers: pp. 1–14

1. Who was Herodotus and what was his relation with Egypt?
2. Describe the important geographic features of Egypt.
3. Why is Egyptian number notation called "base ten"? Why is it said to be "non-positional"?

Do problems 1–4 on page 19.

Simple Egyptian arithmetic: pp. 33–39

1. What were the "Rosetta Stone" and the "Rhind Papyrus"?
2. Describe, and give an example, of Egyptian multiplication by doubling.
3. Describe, and give an example, where there is no remainder in Egyptian division.

Do problems 1, 2a on page 51 (Math majors add rest of #2).

Egyptian fractions: pp. 39–46

1. What are "unit fractions"?

2. What were the rules for the kinds of fractions Egyptians could write?

Do problems 2, 3 on page 52. (Math majors: Describe "Fibonacci's Method" [p. 45] and the technique of "false position." Do problems 12, 13, 16, and 19 on page 51 et.seq.)
Math major project: Prove that every rational number is a sum of distinct unit fractions.

Babylonian numbers: pp. 20–27

1. Approximately what country is Babylonia today?

2. What is "cuneiform" and how was it produced?

3. What does "sexagesimal" mean?

4. Why is the Babylonian system called "positional"?

Do problems 1, 2 on page 29.

Babylonian arithmetic: pp. 20–27

1. Compare Babylonian fractions with Egyptian fractions.

2. Compare Babylonian fractions with *our* fractions.

Do problems 3–5 on page 29.

Babylonian mathematics: pp. 63–70
Do problems 1, 2 on page 72.
(Math majors add: 3, 4, 6 , 12 on page 72.)

II. Geometry in the Ancient World

(You will need a compass for drawing circles.)

Egyptian geometry: pp. 53–61

1. Where does the word "geometry" come from? What does it mean?

2. What kind of geometric formulas did the Egyptian have? Were they accurate? Give examples.

Thales: pp. 85–92
How did Thales' approach to geometry differ from that of the Egyptians?

Pythagoras: pp. 92–99 (Math majors: pp. 92–104); also, pp. 107–109

1. What is the "Pythagorean Theorem"? When was it known?

2. Learn a proof of the Pythagorean Theorem (e.g., the one in the book or on our website).

Do problems 1 and 4 on page 105. (Math majors: do 3, 6 and 7 as well.)

Numbers and magnitudes; Eudoxos: pp. 112–113, 117–119

1. Why was/is the "irrationality" of $\sqrt{2}$ surprising or upsetting?

2. Learn the proof, from the website or the book, that $\sqrt{2}$ is irrational.

Constructions and idealization: pp. 122–129

1. Do the *Construction Assignment* from the website.
2. (Math majors): problem 4, p. 130
3. Read about Plato's Academy, pp. 136–138; compare the Academy to universities of today.

Alexandria and Euclid: pp. 143–149

1. Where is Alexandria and what was its relation to Greece?
2. What is Euclid's *Elements*? "Elements" of what?
3. What are postulates or axioms? How did the "axiomatic system" work? Was it *practical*?
4. Contrast Greek and Egyptian mathematics.

(Math majors: problems 1-4 on page 170)

Greek number theory: pp. 172–176

1. What does it mean to say "a divides b"?
2. What is a prime number? Give some examples. What is a non-prime number called?
3. Learn the proof that "There are an infinite number of primes" (p. 183; revised on our website.)

Do problem 1 on page 184 (math majors: do 5, 6, 10).

Eratosthenes and Ptolemy: pp. 185–194

1. What two things is Eratosthenes most famous for?
2. Explain in your own words how to find all primes less than 50, using Eratosthenes "sieve."
3. What was the *Almagest*?

Math majors: problem 1 on page 194; also, look up a proof of *Brahmagupta's Theorem* (see problem 6).

Archimedes: pp. 196–199 (math majors: 196–208)

1. What were some of Archimedes' most notable mathematical achievements?
2. Archimedes made contributions to physics: use the web to find a description of some of them.
3. Find out about Archimedes' *On the Sphere and the Cylinder*. What is a *palimpsest* and why is the term related to Archimedes?

The decline of Greek mathematics; Diophantus: pp. 215–222

1. How did Roman and early Christian societies differ from Hellenistic society with respect to science and mathematics?
2. Who was Hypatia? (See page 235 and the Web.)
3. What happened to the Library of Alexandria?

III. Early Mathematics in the East

Indian mathematics: pp. 227—230

1. Name some of the main Indian mathematicians and some of their contributions
2. How did their work differ from that of the Greeks?

Arab mathematics: pp. 240–254 (Non-math-majors can skim some of the math.)

1. What was the House of Wisdom?
2. What modern mathematical ideas and terms came from Al-Khowarizmi?
3. Who was Omar Khayyam?

Do problems 1 and 2 on page 266.

Chinese mathematics: pp. 254–256, 260–265

1. In what areas did Chinese mathematicians progress past the Greeks?
2. What sort of mathematical notation did the Chinese contribute?

IV. Decline and Revival in Europe

Decline: pp. 271–276

1. What were the roles of the Church and the Arabs during the "Dark Ages"?
2. How was scientific and mathematical learning affected?

Fibonacci: pp. 279–285, 289–294

1. What was the Hindu-Arabic number system, and how was it related to Fibonacci?
2. What was the significance of *Liber Abaci*?
3. What is the *Fibonacci Sequence?* See the *Fibonacci division of the square* on the website. Do the Fibonacci problem set from the website, and some of problems 1–4 on page 287.

The Renaissance: pp 303–327, 350–357

1. What was the significance of "calendar reform"? Printing? Paper?
2. What were some of the great classical universities founded in the early renaissance?
3. Who were Cardan and Tartaglia?
4. What were the significant contributions of Simon Stevin and John Napier and why were they important?

Math majors: do problems 1 and some part(s) of 3 on page 328; extra credit for problem 2 and/or problem 15.

V. Astronomy

Galileo: pp. 339–347

1. Who was Copernicus?
2. What was the *Inquisition*? (Look this up.) How did Galileo get into trouble with the Italian Inquisition?
3. Read the article about Galileo and the Vatican from the *Globe*, 1992, on our website. What do you think about it?

Kepler: pp. 357–362

1. Who was Tycho?
2. How did Kepler's work improve on the related heliocentric theories of Copernicus and Galileo?
3. What are Kepler's three laws?

Descartes: pp. 364–379

1. How were Descartes' ideas related to Euclid's?
2. What is *Cartesian geometry?*
3. What is the significance of "Cogito ergo sum"?
4. What are "vortices"?

Math majors: do problems 1, 4, 6, 7 on page 381.

VI. The Newtonian Age

Newton, part 1: pp. 382–393

1. Who was Isaac Barrow?
2. What were Newton's 3 major subjects of study during the "plague years?"
3. What is the Lucasian Chair? (Find out who *currently* holds it.)
4. What is the Royal Society?

Newton, part 2: pp. 393–409

1. Who was Edmund Halley?
2. How did Newton feel about Descartes' cosmology?

Read the summary of Newton's Laws on our webpage, and learn them. Do problem 1 on page 409.

(Math majors: (a) Use Barrow's method to find the derivative of a simple polynomial, and compare it to modern limit methods; (b) What were a few of Newton's other mathematical contributions? (c) Compare Newton's and Leibniz' calculus notations.)

Calculus: pp. 410–432

1. Who invented calculus "first"?
2. Who were Maria Agnesi and Emilie du Châtelet?

3. Find out what the "witch of Agnesi" is.

(Math majors: do 1, 4, 5, 9, 12 on page 433.)

VII. Classical mathematics after Newton

Fermat and the Enlightenment: pp. 497–502, 511–524

1. Who was Mersenne?
2. What does Fermat's "little" theorem say?
3. What is Fermat's "last" theorem? (Read about *Andrew Wiles* on the web.)
4. Who was Bishop Berkely? What did he call Newton's *fluxions*?
5. How would you compare 21st-century America with the Enlightenment in Europe?

Euler: pp. 527–535

1. Describe (math majors: in some detail) a few of Euler's contributions to mathematics.
2. What is the number e (named after Euler)?
3. Read about "The Seven Bridges of Königsberg". Why can't they be traversed exactly once in a single round trip?
4. Understand, as best you can, Euler's formula $\cos\theta + i\sin\theta = e^{i\theta}$, and its consequence: $e^{i\pi} = -1$. (This last formula has a lot of pieces of history in it!)

Gauss, the Prince of Mathematics: pp. 539–555

We will discuss congruences and other topics related to Gauss.

Non-Euclidean geometries: pp. 561–569, 583–595.
We will discuss some of these topics.

VIII. Hilbert, Cantor, Gödel and contemporary mathematics

Topics to be announced (as time permits).