

1. The following table gives the gas mileage (in MPG = Miles Per Gallon) for a car at various speeds (in MPH = Miles Per Hour).

MPH:	40	45	50	55	60	65
MPG:	26.1	26.7	27.0	27.2	26.5	25.5

- (a) What is the *average rate of change* of gas mileage with speed between 45 and 55 MPH? (15 pts; you *must give units*.)

$$\text{Avg. R of C} = \frac{\text{Change in MPG}}{\text{Change in MPH}} = \frac{27.2 - 26.7}{55 - 45} = \frac{0.5}{10} = 0.05 \text{ Miles per gallon per miles per hour.}$$

- (b) What is the *average rate of change* of gas mileage with speed between 55 and 65 MPH? (15 pts; you *must give units*.)

$$\text{Avg. R of C} = \frac{\text{Change in MPG}}{\text{Change in MPH}} = \frac{25.5 - 27.2}{65 - 55} = \frac{-1.7}{10} = -0.17 \text{ Miles per gallon per miles per hour.}$$

- (c) In what sense is 55 MPH a “cut off” point with respect to gas mileage and speed? (5 pts)  
Below 55 gas mileage is increasing with speed, while above 55 it decreases with speed.

2. Suppose  $F$  is a function of the variable  $x$ .

- (a) Write down the definition of the *instantaneous rate of change of  $F$*  at the value  $x = a$ . (15 pts)

$$F'(a) = \lim_{h \rightarrow 0} \frac{F(a+h) - F(a)}{h} \text{ or } F'(a) = \lim_{x \rightarrow a} \frac{F(x) - F(a)}{x - a}$$

- (b) Suppose  $F(x) = \frac{3}{x}$ . Use this (limit) definition to find the instantaneous rate of change of  $F$  at  $x = 4$ . (25 pts)

$$\begin{aligned} \frac{F(4+h) - F(4)}{h} &= \frac{\frac{3}{4+h} - \frac{3}{4}}{h} \\ &= \left[ \frac{12 - 3(4+h)}{4(4+h)} \right] \frac{1}{h} \\ &= \left[ \frac{12 - 12 - 3h}{4(4+h)} \right] \frac{1}{h} \\ &= \frac{-3}{4(4+h)} \\ &\rightarrow \frac{-3}{16} \text{ as } h \rightarrow 0. \end{aligned}$$

3. Here's what we know about a function  $H(x)$ :  $H(-2) = 17$  and  $H'(-2) = \frac{8}{5}$ . Use these facts to find the *equation of the tangent line* to the graph of  $y = H(x)$  at the point where  $x = -2$  (25 pts)

The line passes through  $(-2, 17)$  and its slope  $= H'(-2) = \frac{8}{5}$ , so using the point-slope form of the equation of a line,  $y - 17 = \frac{8}{5}(x + 2)$ , or  $y = \frac{8}{5}(x + 2) + 17$ .