MTH 241 Quiz 4 Answers

1. The left-hand plot below is of $y = f'(x)$ — i.e. the *derivative* of f. Use it to answer the following questions. (12 pts each)

   (a) Approximately where (give intervals) is the function $f(x)$ *increasing?*

   $f(x)$ is increasing where its derivative $f'(x)$ *is positive.* The graph on the left suggests that this is in the intervals $-8 \le x \le 0$ and $x \ge 4$.

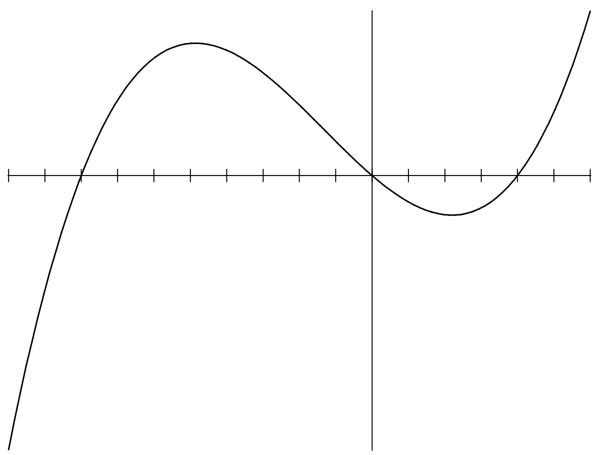
   (b) At which values of x does the graph of $y = f(x)$ (*not the graph shown*) have *horizontal tangents?*

   Horizontal tangents occur when the derivative is 0; that is, when the graph of $f'(x)$ crosses the x-axis. This occurs at $x = -8$, $x = 0$ and $x = 4$.

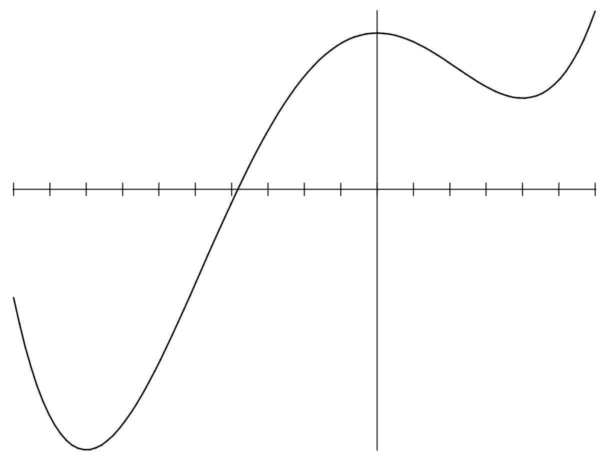
   (c) Approximately where (give interval) is the graph of $y = f(x)$ *concave down?*

   The graph of a function is concave down when the second derivative is negative. Since the second derivative measures the rate of change of the first derivative, the second derivative will be negative when the first derivative is *decreasing*. Looking at the graph of $f'(x)$, this appears to be on the interval $-5 \le x \le 2$

   (d) On the axes to the right, sketch a possible graph of $y = f(x)$.



Graph of $y = f'(x)$



A possible graph of $y = f(x)$

In case you are interested, the actual function I used here is $f(x) = \frac{x^4}{4} + \frac{x^3}{3} - x^2 + 1$, so $f'(x) = x^3 + x^2 - 2x$.

2. Find the derivatives of the following functions. (13 pts each)

   (a) $T(x) = \sqrt{x} - \frac{3}{x} + 4x^{2/3}$

   $T(x) = x^{1/2} - 3x^{-1} + 4x^{2/3}$ so

   $T'(x) = \frac{1}{2}x^{-1/2} + 3x^{-2} + \frac{8}{3}x^{-1/3}$.

   (b) $H(u) = (u^3 - 2u + 7)e^u$

   We use the product rule here:

   $H'(u) = (u^3 - 2u + 7)e^u + (3u^2 - 2)e^u = (u^3 + 3u^2 - 2u + 5)e^u$. (Either answer is correct — never multiply these out!)

(c) $Q(t) = \dfrac{1+t+t^2}{1-t-t^2}$

We use the quotient rule here:

$$Q'(t) = \frac{(1-t-t^2)(1+2t) - (1+t+t^2)(-1-2t)}{(1-t-t^2)^2}.$$

3. The tangent line to the graph of $y = M(x)$ at $(3,9)$ passes through the point $(1,4)$. Find the value of $M'(3)$? (13 pts)

$M'(3)$ is the slope of the tangent line at $x = 3$. Since this line passes through $(3,9)$ and $(1,4)$ (given), its slope is $\dfrac{9-4}{3-1} = \dfrac{5}{2}$. Thus, $M'(3) = 5/2$.