MTH U241 Quiz 5 Answers

1. Find the derivatives of these functions. You do not have to simplify (12 pts each).

   (a) $(3x^3 - 4x^2 + 2x + 1)^{99}$ (Chain rule)
   $99(3x^3 - 4x^2 + 2x + 1)^{98}(9x^2 - 8x + 2)$

   (b) $\dfrac{\sin(x^2+1)}{\cos(x)}$ (Quotient rule + chain rule)
   $\dfrac{\cos(x)\cos(x^2+1)2x - \sin(x^2+1)(-\sin(x))}{\cos^2(x)}$

   (c) $e^{\tan x}$ (Chain rule)
   $e^{\tan x}\sec^2 x$

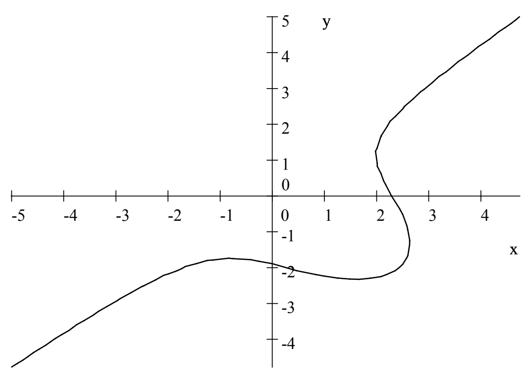
   (d) $\sin^2(x^6)$ (Chain rule twice)
   $\underbrace{2\sin(x^6)}_{\text{deriv of ( )}^2} \cdot \underbrace{(\cos(x^6)}_{\text{deriv of sin}} \cdot \underbrace{6x^5}_{\text{deriv of } x^6}$

2. Suppose that $f(3) = 2$ and $f'(3) = 7$, and let $H(x) = \sqrt{8 + f(x)}$. Find $H'(3)$. (Chain rule, 12 pts)

   $H'(x) = \frac{1}{2}(8 + f(x))^{-1/2} \cdot f'(x)$, so $H'(3) = \frac{1}{2}(8 + f(3))^{-1/2} \cdot f'(3) = \frac{1}{2}(8+2)^{-1/2} \cdot 7 = \dfrac{7}{2\sqrt{10}}$.

3. The equation $x^3 - y^3 + 2xy = x^2 + 7$ defines a curve $\mathcal{C}$ and also *implicitly* determines y as a function of x.

   (a) Verify that the point $(2, 1)$ lies on $\mathcal{C}$. (5 pts)
   The left-hand side is $2^3 - 1^3 + 2(2)(1) = 11$; the right-hand side is $2^2 + 7 = 11$, so they are equal.



   (b) Find the *slope of the tangent line* to $\mathcal{C}$ at the point $(2, 1)$ using implicit differentiation. (35 pts)
   Differentiating both sides gives:

$$\begin{aligned} 3x^2 - 3y^2y' + (2xy' + 2y) &= 2x \\ -3y^2y' + 2xy' &= 2x - 3x^2 - 2y \\ (2x - 3y^2)y' &= 2x - 3x^2 - 2y \\ y' &= \frac{2x - 3x^2 - 2y}{2x - 3y^2} \end{aligned}$$

   When $x = 2$ and $y = 1$ this gives $y' = \dfrac{4 - 12 - 2}{4 - 3} = \dfrac{-10}{1} = -10$.