1. Find the derivatives of the following functions (12 pts each).

   (a) $\ln(x^3 - x^2 + 10)$

   $\dfrac{3x^2 - 2x}{x^3 - x^2 + 10}$

   (b) $\sin(x + \ln x)$

   $\cos(x + \ln x) \cdot \left(1 + \dfrac{1}{x}\right)$

   (c) $\arctan(\sqrt{3x+2})$

   $$\underbrace{\frac{1}{1 + \left(\sqrt{3x+2}\right)^2} \cdot \frac{1}{2\sqrt{3x+2}} \cdot 3}_{\text{Perfectly correct as is}} = \frac{1}{2\,(x+1)\,\sqrt{3x+2}}$$

2. Suppose $y = \dfrac{(\sin x)(x-1)}{(x^3+2)(x+3)}$. Use logarithmic differentiation to find y'. (20 pts)

   Take logs of both sides: $\ln y = \ln(\sin x) + \ln(x-1) - \ln(x^3+2) - \ln(x+3)$

   Differentiate: $\dfrac{y'}{y} = \dfrac{\cos x}{\sin x} + \dfrac{1}{x-1} - \dfrac{3x^2}{x^3+2} - \dfrac{1}{x+3}$

   $$y' = \frac{(\sin x)(x-1)}{(x^3+2)(x+3)} \left(\frac{\cos x}{\sin x} + \frac{1}{x-1} - \frac{3x^2}{x^3+2} - \frac{1}{x+3} \right)$$

3. You want to find the *area of a disk* ($A = \pi R^2$) by measuring its *circumference* ($C = 2\pi R$). Using a tape measure, you find C to be 100 cm with an error of at most 0.5 cm. Use the tangent-line approximation to estimate the error in computing A. (Hint: First write A in terms of C.) Now, compute the *percentage* or *relative* error $\dfrac{\Delta A}{A}$ as well. (24 pts)

   $A = \pi R^2$ and $C = 2\pi R$, so $A = \pi \left(\dfrac{C}{2\pi}\right)^2 = \dfrac{\pi C^2}{4\pi^2} = \dfrac{C^2}{4\pi}$. The "tangent-line approximation" says that $\Delta A \approx \dfrac{dA}{dC}\Delta C$, so we compute $\dfrac{dA}{dC} = \dfrac{2C}{4\pi} = \dfrac{C}{2\pi}$. Putting this in the approximation gives $\Delta A \approx \left(\dfrac{C}{2\pi}\right)\Delta C$. We are *given*: $C = 100$ and $\Delta C = \dfrac{1}{2}$ so

   $$\Delta A \approx \frac{100}{2\pi} \cdot \frac{1}{2} = \frac{25}{\pi} \approx 7.96.$$

   Also, we compute the *relative error*:

   $$\frac{\Delta A}{A} \approx \frac{25/\pi}{100^2/(4\pi)} = \frac{1}{100} \quad (= 1\%).$$

4. Suppose I know that $f(9) = 20$ and $f'(9) = \dfrac{1}{4}$. Use the *best linear or tangent line approximation* to f to estimate $f(8.84)$. (20 pts)

   $$\begin{aligned} f(8.84) &\approx f(9) + f'(9)(8.84 - 9.00) \\ &= 20 + (1/4)(-0.16) \\ &= 19.96. \end{aligned}$$