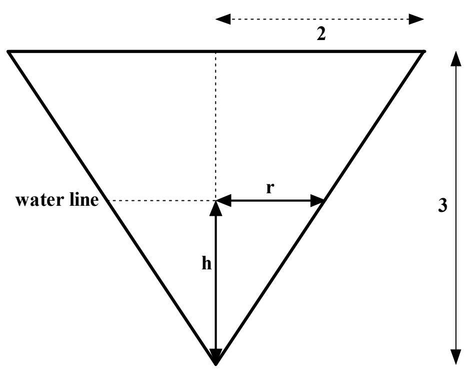
1. A tank is in the shape of an inverted (point downward) cone. The height of the cone is 3 meters and the radius of its base is 2 meters. Water is draining from the tank (at the pointed end) at a rate of 1/10 cu. m per second. How fast is the water level decreasing when the level is 1 meter? (For a cone, $V = \frac{1}{3}\pi r^2 h$.)



We have to relate the volume V to the water level h. In the formula $V = \frac{1}{3}\pi r^2 h$ we must write the radius r in terms of h. Using similar triangles (see diagram) we have: $\frac{r}{2} = \frac{h}{3}$, or $r = \frac{2h}{3}$. Substituting this into the volume formula gives: $V = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi}{27}h^3$. We now differentiate with respect to time t: $\frac{dV}{dt} = \frac{4\pi}{9}h^2\frac{dh}{dt}$. We want $h = 1$, and we are given $\frac{dV}{dt} = -1/10$. This gives the equation $-1/10 = \frac{4\pi}{9}(1)^2\frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{-9}{40\pi}$, or approximately $-.072$ m/s.

2. Use calculus to find the absolute maximum and absolute minimum of $f(x) = x^3 - 3x^2 + 5$ on the interval $[-1, 4]$.

$f'(x) = 3x^2 - 6x = 3x(x-2) \overset{\text{set}}{=} 0$ so we get two critical numbers: $x = 0$, $f(0) = 5$ and $x = 2$, $f(2) = 1$. We also check the endpoints: $f(-1) = 1$, and $f(4) = 21$. So the maximim value is 21 occurring at the endpoint $x = 4$, and the minimum value is 1, occurring at both the critical number $x = 2$ and the endpoint $x = -1$. Here is a plot: