1. Suppose $g(x) = x^4 - 4x^3$

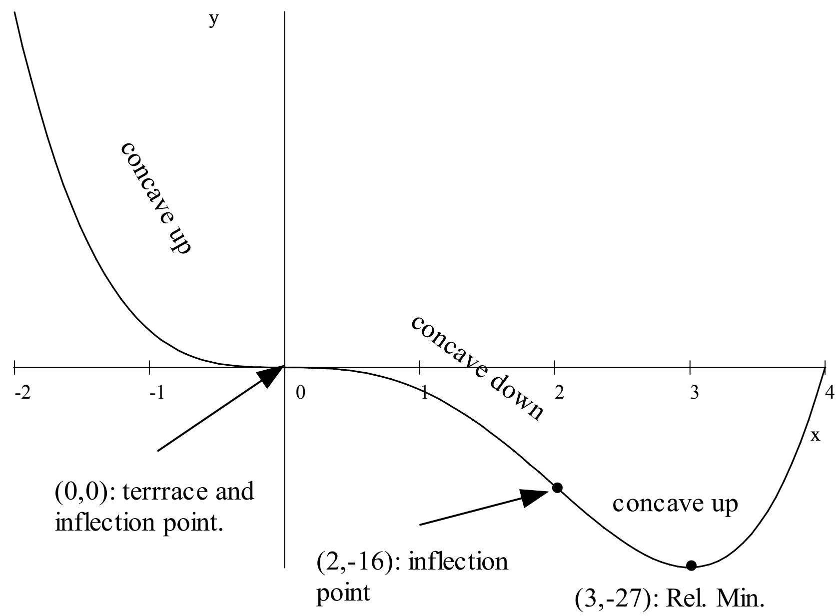
(a) Find all critical points for g and determine which are maxima/minima or terrace points.

$g'(x) = 4x^3 - 12x^2 = 4x^2(x-3) \overset{\text{set}}{=} 0$. This gives $x = 0$ and $x = 3$ as critical numbers; the corresponding y-values are 0 and -27, so we get the points $(0,0)$ and $(3,-27)$. Checking to the left of 0, $g'(-10)$ (say) is < 0, so g is decreasing for $x < 0$. Since $g'(1)$ (say) is also < 0, we see that g is also decreasing for $0 < x < 3$. Since $g'(10) > 0$, g is increasing for $x > 0$. Since g is decreasing on either side of 0, $x = 0$ is neither a max nor a min: it is a terrace point. On the other hand, g is decreasing to the left of 3 and increasing to the right, so $x = 3$ is a relative minimum. (You could also use the second derivative test for $x = 3$.)

(b) Find any inflection points for g.

$g''(x) = 12x^2 - 24x = 12x(x-2)$. For $x < 0$, $g''(x)$ is positive; for $0 < x < 2$, $g''(x) < 0$, and for $x > 2$, $g''(x) > 0$. Thus, the concavity changes at both of these points, so $(0,0)$ and $(2,-16)$ are inflection points.

(c) Draw a sketch of the graph of g, labeling the points found above (you may use your calculator). $x^4 - 4x^3$



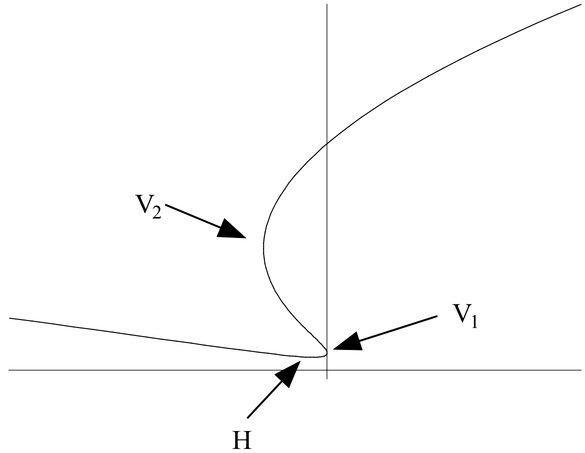
2.

Consider the parametric curve:

$$x = t^3 - 3t^2$$
$$y = t^2 + t + 1$$

(plot is to the right).

Note: $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{3t^2 - 6t}$$



(a) Find the value(s) of t for which there is a *horizontal tangent,* then find the x, y coordinates of the corresponding points. Indicate them in the plot above.

The tangent will be horizontal when $\frac{dy}{dx} = 0$, i.e. when $2t + 1 = 0$, $t = -1/2$. This is the point $H = (-7/8, 3/4)$.

(b) Do the same as part (a) except for *vertical tangents.*

The tangent will be vertical when $3t^2 - 6t = 3t(t-2) = 0$, i.e. when $t = 0$ and $t = 2$. These correspond to the points $V_1 = (0, 1)$, and $V_2 = (-4, 7)$.