What You Need to Know for the Final

Here is a list of things you should be able to do for the final; all work must be shown.

1. Solve a general first-order linear equation $\frac{dy}{dx} + P(x)y = Q(x)$ using an integrating factor.

2. Use the Laplace transform to find solutions to initial value linear, constant coefficient first order equations of the form $a\frac{dy}{dx} + by = Q(x)$, where $Q(x)$ is a simple function.

3. Analyze the solutions to an autonomous differential equation without actually solving it, by finding equilibrium solutions and using the sign of first and second derivatives to sketch the solution curves.

4. For a constant coefficient second order homogeneous equation, find the complete solution (two linearly independent solutions). Given initial values, find the specific solution satisfying them.

5. Use the method of undetermined coefficients to find the solution of a second order constant coefficient non-homogeneous problem: $(aD^2 + bD + c)y = f(x)$.

6. Use the Laplace transform to find the solution of a constant coefficient second order equation with a driving force term equal to (a) a simple exponential or trigonometric function, or (b) a step function or (c) a delta function.

7. Given a system of linear equations, (a) write them in matrix form $A \cdot \mathrm{x} = \mathrm{b}$; (b) solve the system (by hand or by using a calculator to find the row reduced echelon form); (c) write the general solution as the general solution of $A \cdot \mathrm{x} = 0$ plus a particular solution of the non-homogeneous equation $A \cdot \mathrm{x} = \mathrm{b}$; (d) find a basis for the solution space of $A \cdot \mathrm{x} = 0$.

8. Given vectors $\mathbf{v_1}$, $\mathbf{v_2}$, …, $\mathbf{v_k}$, (a) be able to determine if they are independent, (b) be able to determine is a given vector **w** is in their span, and (c) be able to extract from them a basis for their span. You should know that the dimension of a vector space is the size of any basis.

9. Given a system of linear first order equations, (a) write it in matrix form $\mathrm{x}' = A\mathrm{x}$; (b) find the eigenvalues and eigenspaces for the matrix A; use these eigenvalues and eigenvectors to find all solutions.

10. Be able to interpret a second-order differential equation in physical terms (e.g. as a spring with or without damping).

11. Be able to compute the Laplace and inverse Laplace transforms of simple functions, including piecewise continuous and impulse (delta) functions. A table of basic Laplace transforms will be provided on the final.