Professor M. Bridger

MTH U343 QUIZ 1 Name:

(You must show all work to get credit. Always give appropriate units.)

1. Solve the following separable initial-value problem: $\frac{dy}{dx} = \frac{2x(y+1)}{1+x^2}$, $y = 19$ when $x = 2$. (30 pts).

$$\begin{aligned} \int \frac{1}{y+1}\,dy &= \int \frac{2x}{1+x^2}\,dx \\ \ln(y+1) &= \ln(1+x^2) + C \\ y+1 &= e^{\ln(1+x^2)+C} = (1+x^2)e^C = K(1+x^2) \end{aligned}$$

Putting in 19 for y and 2 for x gives $20 = K \cdot 5$, so $K = 4$ and

$$y = 4(1+x^2) - 1.$$

2. Solve the following linear initial-value problem: $\frac{dy}{dx} + \frac{3y}{x} = 6x^2 + 5x$, $y(2) = 13$. (30 pts)

Theintegrating factor is: $\rho = e^{\int (3/x)\ dx} = e^{3\ln(x)} = x^3$. Multiplying by ρ gives

$$x^3\frac{dy}{dx} + 3x^2y = 6x^5 + 5x^4.$$

Thus, $\frac{d}{dx}(x^3y) = 6x^5 + 5x^4$, so $x^3y = x^6 + x^5 + C$; i.e. $y = x^3 + x^2 + \frac{C}{x^3}$. The initial condition gives

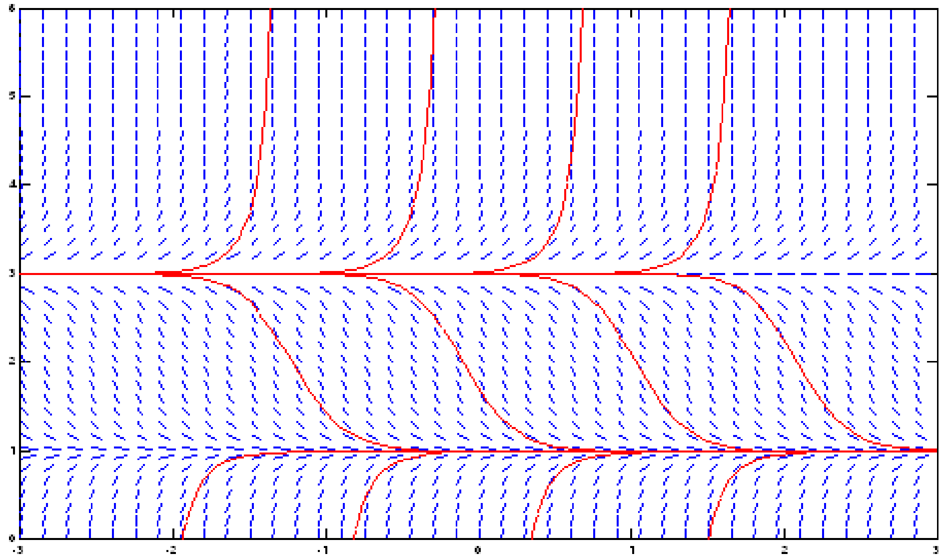
$$\begin{aligned} 13 &= 8 + 4 + C/8, \text{ or } C = 8. \\ y &= x^3 + x^2 + \frac{8}{x^3}. \end{aligned}$$

3. Consider the autonomous differential equation $\frac{dy}{dx} = 2(y-1)(y-3)$.

   (a) Draw a phase line indicating critical points and where y is is incr/decr (derivative positive) (10 pts)

   Critical points: $y = 1$ and $y = 3$. $\frac{dy}{dx}$ is positive (y increasing) for $y < 1$. $\frac{dy}{dx} < 0$ (y decreasing) for $1 < y < 3$; $\frac{dy}{dx} > 0$ again for $y > 3$.

   (b) Draw a sketch of some representative solution curves. Make sure you include the equilibrium solutions, labelling which are *stable* and *unstable*. (30 pts)



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