1. An object moves horizontally through a medium in which resistance is proportional to v^2; in fact, $\frac{dv}{dt} = -3v^2$. Suppose we also know that its initial ($t = 0$) velocity $v_0 = 5$ and its initial position is $x_0 = 0$.

   (a) Find its velocity v at time t. (Separating variables) (20 pts)

$$\int \frac{dv}{v^2} = \int -3\,dt, \text{ so } -\frac{1}{v} = -3t + C. \text{ The initial condition gives } C = -\frac{1}{5}.$$
$$v = \frac{1}{3t + 1/5}.$$

   (b) Find its position x at time t. (20 pts)

$$\frac{dx}{dt} = v = \frac{1}{3t + 1/5}$$
$$x = \int \frac{dt}{3t + 1/5} = \frac{1}{3}\ln(3t + 1/5) + C$$

   The initial condition gives $C = -\frac{1}{3}\ln(1/5)$.

$$x = \frac{1}{3}\ln(3t + 1/5) - \frac{1}{3}\ln(1/5) = \frac{1}{3}\ln(15t + 1).$$

2. Find the Laplace transform $H(s)$ of the following functions. (A table is printed on the reverse side; also, facts about the Gamma function.) (12 pts each)

   (a) $h(t) = 7 - e^{4t} + 6e^{-5t}$; $H(s) = \frac{7}{s} - \frac{1}{s-4} + \frac{6}{s+5}$

   (b) $h(t) = 3\cos(4t) - 2\sin(t)$; $H(s) = \frac{3s}{s^2+16} - \frac{2}{s^2+1}$

   (c) $h(t) = 3t^{3/2}$; $H(s) = 3 \cdot \frac{\Gamma(5/2)}{s^{5/2}} = 3 \cdot \frac{(3/2)\Gamma(3/2)}{s^{5/2}} = 3 \cdot \frac{(3/2)(1/2)\Gamma(1/2)}{s^{5/2}} = \frac{(9/4)\sqrt{\pi}}{s^{5/2}}$

3. Find the *inverse* Laplace transform $h(t)$ of each of the following. (12 pts each)

   (a) $H(s) = \frac{5}{s} - \frac{3}{s^2} + \frac{1}{s^7}$; from the table $\left(\mathcal{L}^{-1}\left\{\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}\right\} = t^\alpha\right.$, so:

$$\begin{aligned} h(t) &= 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^7}\right\} \\ &= 5 - 3t + \mathcal{L}^{-1}\left\{\frac{1}{\Gamma(7)} \cdot \frac{\Gamma(7)}{s^7}\right\} \\ &= 5 - 3t + \frac{1}{\Gamma(7)}\mathcal{L}^{-1}\left\{\frac{\Gamma(7)}{s^7}\right\} \\ &= 5 - 3t + \frac{1}{6!} \cdot t^6 = 5 - 3t + \frac{t^6}{720}. \end{aligned}$$

   (b) $H(s) = \frac{1}{s-1} + \frac{2}{s+2} - \frac{3}{s-3}$; from the table:

$$h(t) = e^t + 2e^{-2t} - 3e^{3t}.$$