Professor M. Bridger

MTH U343 QUIZ 3 Name:

(You must show all work to get credit. Always give appropriate units.)

1. Use the Laplace transform to solve the initial value problem $x' - 3x = e^{5t}$, $x(0) = 1$.

$$\begin{aligned}(sX-1)-3X &= \frac{1}{s-5}, \text{ so } X(s-3) = \frac{1}{s-5}+1 = \frac{s-4}{s-5}.\\ X &= \frac{s-4}{(s-3)(s-5)} = \frac{A}{s-3}+\frac{B}{s-5}.\\ s-4 &= A(s-5)+B(s-3).\end{aligned}$$

Letting $s = 5$ gives $B = \frac{1}{2}$ and letting $s = 3$ gives $A = \frac{1}{2}$ also. Thus

$$\begin{aligned}X &= \frac{1/2}{s-3}+\frac{1/2}{s-5}\\ x(t) &= \frac{1}{2}e^{3t}+\frac{1}{2}e^{5t}\end{aligned}$$

2. Use the Laplace transform to solve the initial value problem $x' - 3x = 2t$, $x(0) = 0$.

$$\begin{aligned}sX-3X &= \frac{2}{s^2} \text{ so } X = \frac{2}{s^2(s-3)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{x-3}\\ 2 &= As(s-3)+B(s-3)+Cs^2.\end{aligned}$$

Letting $s = 0$ gives $B = -\frac{2}{3}$ and letting $s = 3$ gives $C = \frac{2}{9}$. Now let $s = 1$, so $2 = -2A - 2(-2/3) + 2/9$ and $A = -\frac{2}{9}$. Thus

$$\begin{aligned}X &= \frac{-2/9}{s}-\frac{2/3}{s^2}+\frac{2/9}{s-3}\\ x(t) &= -\frac{2}{9}-\frac{2}{3}t+\frac{2}{9}e^{3t}.\end{aligned}$$

3. Convert to following equations into matrix form and row-reduce. Give the general solution and *any two* particular solutions.

$$\begin{aligned}x \quad\quad - 3z &= 2\\ 4y+8z &= -4\\ 2x+4y+2z &= 0.\end{aligned}$$

$$\left(\begin{array}{ccc|c}1&0&-3&2\\0&4&8&-4\\2&4&2&0\end{array}\right)\longrightarrow\left(\begin{array}{ccc|c}1&0&-3&2\\0&4&8&-4\\0&4&8&4\end{array}\right)\longrightarrow\left(\begin{array}{ccc|c}1&0&-3&2\\0&1&2&-1\\0&0&0&0\end{array}\right)$$

We read off the solution (letting the free variable $z = t$):

$$x = 2+3t, y = -1-2t, z = t.$$

Letting $t = 0$ gives $\left\{\begin{array}{c}x=2\\y=-1\\z=0\end{array}\right.$; letting $t = 1$ gives $\left\{\begin{array}{c}x=5\\y=-3\\z=1\end{array}\right.$; letting $t = 2$ gives $\left\{\begin{array}{c}x=8\\y=-5\\z=2\end{array}\right.$; letting $t = -1$ gives $\left\{\begin{array}{c}x=-1\\y=1\\z=-1\end{array}\right.$.