(You must show all work to get credit. Always give appropriate units.)

1. The system of equations $\left\{ \begin{array}{l} x - 4y + 3z - 4w = 10 \\ x - 3y + 3z - 6w = 11 \\ 2x - 8y + 7z - 13w = 24 \end{array} \right\}$ is written in matrix form and row-reduced, obtaining (take my word for it) the following augmented matrix:

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 2 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -5 & 4 \end{array} \right).$$

Write the general solution in *vector form*, and use it to find *two* different solutions to the original equation.

x, y and z are pivot variables, but w is free. So we can let w be anything, say s; then:

$\begin{array}{rcccc} x & = & 2 - 3w & = & 2 - 3s \\ y & = & 1 + 2w & = & 1 + 2s \\ z & = & 4 + 5w & = & 4 + 5s \\ w & = & 0 + w & = & 0 + s \end{array}$. Thus $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \\ 0 \end{pmatrix} + s \begin{pmatrix} -3 \\ 2 \\ 5 \\ 1 \end{pmatrix}$ is the general solution. s can be any number, so choose the simple values $s = 0$ and $s = 1$, which give the solutions

$$\begin{pmatrix} 2 \\ 1 \\ 4 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ 3 \\ 9 \\ 1 \end{pmatrix} \text{ (there are, of course, infinitely many other correct answers).}$$

2. Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & 4 \\ 1 & 7 & 2 \\ 0 & -2 & 5 \end{pmatrix}$. Let $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ be the 3 column vectors of A. Calculate the linear combination $2\mathbf{v}_1 - 3\mathbf{v}_2 + \mathbf{v}_3$.

$$= 2 \begin{pmatrix} 2 \\ 3 \\ 1 \\ 0 \end{pmatrix} - 3 \begin{pmatrix} -1 \\ 1 \\ 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 3 \\ 21 \\ -6 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ -17 \\ 11 \end{pmatrix}.$$

3. Suppose U is a 9×4 matrix and V is a $k \times m$ matrix.

   (a) When will the product UV be defined?
   When $k = 4$.

   (b) What will be its dimensions?
   $9 \times m$

   (c) When will the product VU be defined?
   When $m = 9$.

   (d) What will be its dimensions?
   $k \times 4$.

4. Calculate the matrix product $\begin{pmatrix} 1 & 4 & 0 \\ -1 & 2 & 1 \\ 5 & 0 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 3 \\ -2 & 3 & 3 & 0 \\ 5 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 14 & 15 & 3 \\ 0 & 5 & 5 & -2 \\ 35 & 16 & 27 & 21 \end{pmatrix}$