1. Use row-reduction to find the inverse of the matrix $\begin{pmatrix}1&2&0\\0&1&0\\1&5&1\end{pmatrix}$.

$$\begin{pmatrix}1&2&0&1&0&0\\0&1&0&0&1&0\\1&5&1&0&0&1\end{pmatrix}\longrightarrow\begin{pmatrix}1&2&0&1&0&0\\0&1&0&0&1&0\\0&3&1&-1&0&1\end{pmatrix}\longrightarrow\begin{pmatrix}1&0&0&1&-2&0\\0&1&0&0&1&0\\0&3&1&-1&0&1\end{pmatrix}$$

$$\longrightarrow\begin{pmatrix}1&0&0&1&-2&0\\0&1&0&0&1&0\\0&0&1&-1&-3&1\end{pmatrix}$$

so $\begin{pmatrix}1&2&0\\0&1&0\\1&5&1\end{pmatrix}^{-1}=\begin{pmatrix}1&-2&0\\0&1&0\\-1&-3&1\end{pmatrix}$

2. Let $A=\begin{pmatrix}2&3\\4&7\end{pmatrix}$. Find the inverse of A and use it to solve the matrix equation $A\cdot X=\begin{pmatrix}-1\\5\end{pmatrix}$.

$A^{-1}=\frac{1}{2}\begin{pmatrix}7&-3\\-4&2\end{pmatrix}$, and

$$X=A^{-1}\cdot\begin{pmatrix}-1\\5\end{pmatrix}=\frac{1}{2}\begin{pmatrix}7&-3\\-4&2\end{pmatrix}\cdot\begin{pmatrix}-1\\5\end{pmatrix}=\frac{1}{2}\begin{pmatrix}-22\\14\end{pmatrix}=\begin{pmatrix}-11\\7\end{pmatrix}.$$

3. Calculate the determinant of the matrix $\begin{pmatrix}1&3&2\\4&0&1\\1&5&5\end{pmatrix}$.

$$\det\begin{pmatrix}1&3&2\\4&0&1\\1&5&5\end{pmatrix}=1\cdot\begin{vmatrix}0&1\\5&5\end{vmatrix}-3\cdot\begin{vmatrix}4&1\\1&5\end{vmatrix}+2\cdot\begin{vmatrix}4&0\\1&5\end{vmatrix}=1\cdot(-5)-3\cdot19+2\cdot20=-22.$$

4. Row reduction is performed on the matrix $A=\begin{pmatrix}2&2&5\\2&2&9\\4&7&11\end{pmatrix}$ to get the matrix $\begin{pmatrix}2&2&5\\0&3&1\\0&0&4\end{pmatrix}$. The only row ops used are adding multiples of one row to another, and exactly one row interchange.

   (a) How are $\det(A)$ and $\det(B)$ related?

   Adding multiples of a row to another doesn't change the determinent, but a row interchange multiplies it by -1; thus $\det(A)=-\det(B)$.

   (b) Calculate $\det(B)$ quickly.

   B is upper triangular, so its determinent is simply the product of its diagonal entries: $\det(B)=24$.