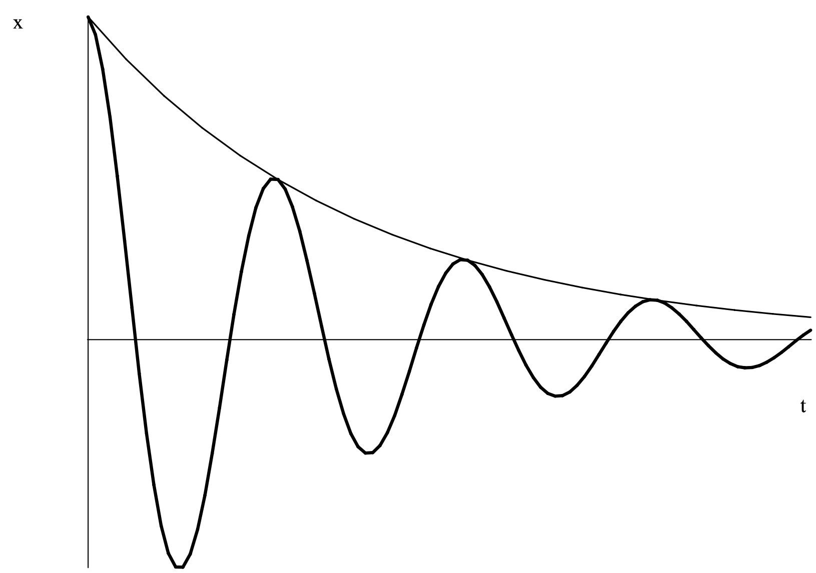
(You must show all work to get credit. Always give appropriate units.)

1. A mass of 3 kilograms is suspended from a spring with spring constant $k = 15$. Assume there is *no damping* and *no external force.*

   (a) Write the differential equation of motion for the mass and find the general solution.
   The differential equation is $3x'' + 15x = 0$, or, equivalently, $x'' + 5x = 0$. The complementary equation is $D^2 + 5 = 0$, with roots $\pm\sqrt{5}i$. This gives the general solution $A \sin\left(\sqrt{5}t\right) + B \cos\left(\sqrt{5}t\right)$.

   (b) Without damping, the motion is simple oscillatory. Draw a sketch of the motion (x versus t) assuming that there is a *small* amount of damping.

The damping causes a gradual decay in amplitude of the form e^{-at}:



2. Let $\mathcal{D}$ be the differential equation $x'' + 2x' + 2x = 10 \cos t$.

   (a) Find the *general solution* to the associated homogeneous equation $x'' + 2x' + 2x = 0$.
   Associated polynomial equation is $D^2 + 2D + 2 = 0$ with roots $-1 \pm i$ (quadratic formula). The general solution is then

$$x_h(t) = e^{-t}(c_1 \cos t + c_2 \sin t)$$

   (b) Using undetermined coefficients, find a *particular* solution to $\mathcal{D}$.
   Since $\cos t$ is not a root of the homogen. eq., we guess $x_p(t) = A \cos t + B \sin t$. so we get:

$$\begin{aligned} x &= A\cos t + B\sin t & 2x &= 2A\cos t + 2B\sin t \\ x' &= B\cos t - A\sin t & 2x' &= 2B\cos t - 2A\sin t \\ x'' &= -A\cos t - B\sin t & x'' &= -A\cos t - B\sin t \\ & & x'' + 2x' + 2x = & \ (A + 2B)\cos t + (B - 2A)\sin t = 10\cos t \end{aligned}$$

   Equating coefficients we get $B - 2A = 0$ and $A + 2B = 10$. Thus, $A = 2$, $B = 4$, so

$$x_p(t) = 2\cos t + 4\sin t.$$

(c) The function $A\cos\omega t + B\sin\omega t$ can be written $C\cos(\omega t - \alpha)$, where $C = \sqrt{A^2+B^2}$, $\cos\alpha = A/C$, and $\sin\alpha = B/C$. Do this with your answer to part (b).
$C = \sqrt{2^2+4^2} = \sqrt{20}$; since A, B are positive, we're in the first quadrant; $\sin\alpha = 4/\sqrt{20}$ so $\alpha \approx 1.10715$ and

$$x_p(t) \approx \sqrt{20}\cos(t - 1.10715).$$

(d) Write down the general solution to $\mathcal{D}$.

$$\begin{aligned} x_g(t) &= x_h(t) + x_p(t) = x_h\,(t) \\ &= \left\{ \begin{array}{l} e^{-t}\,(c_1\cos t + c_2\sin t) + 2\cos t + 4\sin t \\ e^{-t}\,(c_1\cos t + c_2\sin t) + \sqrt{20}\cos(t-\alpha). \end{array} \right\} \text{ (either is correct).} \end{aligned}$$