

Scores by problem:

1	2	3	4	5	6	7	8	9	10	Total
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Instructor: _____

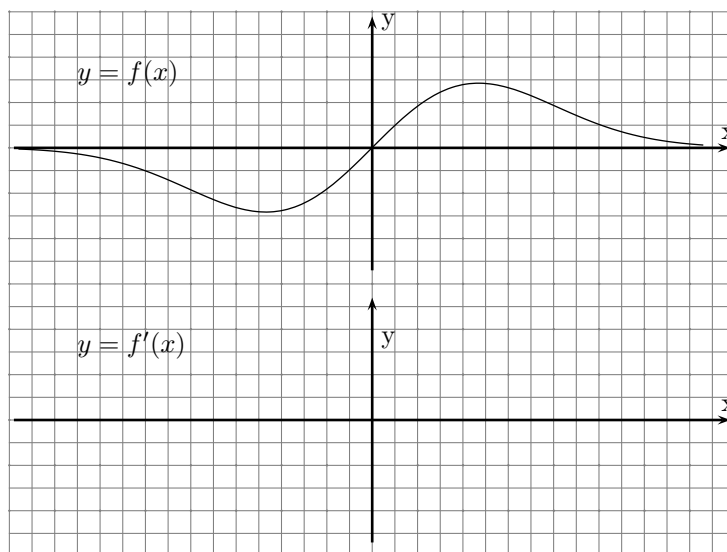
Name: _____

NORTHEASTERN UNIVERSITY
MTH U241 (Calculus I) Final Exam Fall 2008

SHOW YOUR WORK. There is **NO** credit for the answer without the work shown. If there is not enough room to show your work, use the back of the preceding page. Where a blank is provided for your answer, you must place your final answer in that blank to receive credit.

1. (a) For the function $f(x) = \frac{1}{2x+1}$ use the definition of the derivative to write $f'(0)$ as the limit of an algebraic expression, and compute the limit by simplifying this algebraic expression. **(5 pts)**

(b) The graph of function f is given below. Use it to sketch the graph of its derivative f' . On the graph of f indicate (by thickening the corresponding part of the graph) the points where the graph of f is concave upwards. **(5 pts)**



2. Calculate the following derivatives. If possible, simplify your answer.

(a) **(2 pts)** $\frac{d}{du} \left[\frac{u \tan u + u^2}{u} \right] =$ _____

(b) **(2 pts)** $\frac{d}{dx} \left[\frac{\sin x}{\sin x + 1} \right] =$ _____

(c) **(3 pts)** $\frac{d}{dt} \left[\arctan(2t) - \ln(4t^2 + 1) \right] =$ _____

(d) **(3 pts)** $\frac{d}{dx} \left[x^{1/x} \right] =$ _____

3. Consider the curve defined by: $x^3 + y^3 = 2 \ln y$.

(a) Use implicit differentiation to find the expression for dy/dx as a function of x and y . **(3 pts)**

Answer: $dy/dx =$ _____

(b) Find the equation of the tangent line to the curve at $(-1, 1)$. **(4 pts)**

The tangent line is: _____

(c) Use the linear approximation of y at $x = -1$ to estimate the y -coordinate of the point on the curve with x -coordinate being -1.1 . **(3 pts)**

Answer: $y(-1.1) \approx$ _____

4. For the function $y = f(x) = x^2e^x$

(a) Find all critical numbers for $f(x)$. For each critical number determine whether it is a local maximum, local minimum, or none of the above. (**3 pts**)

The critical points, with their types, are

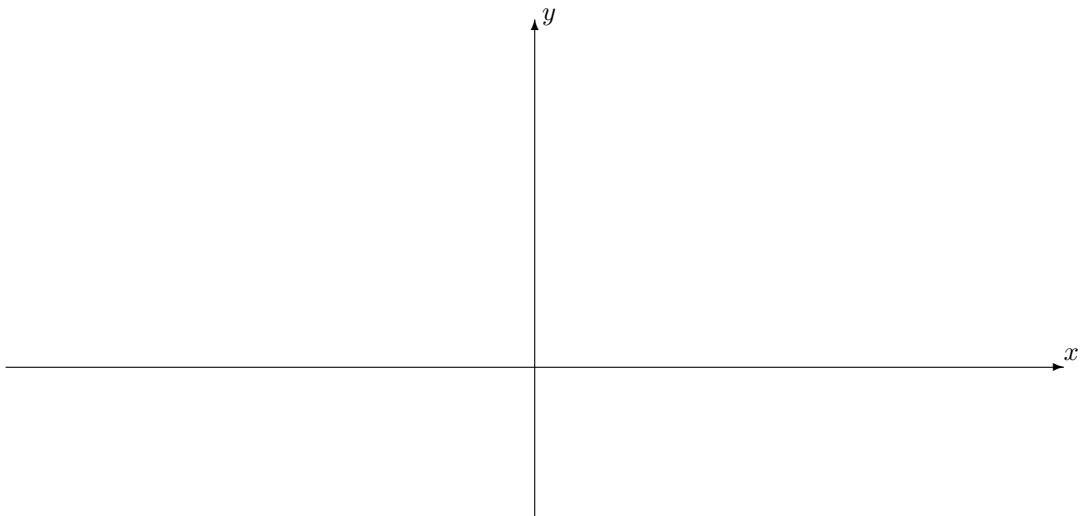
(b) Find intervals of increase/decrease for $f(x)$. (**2 pts**)

Make sure your answer is completely clear, complete, and unambiguous.

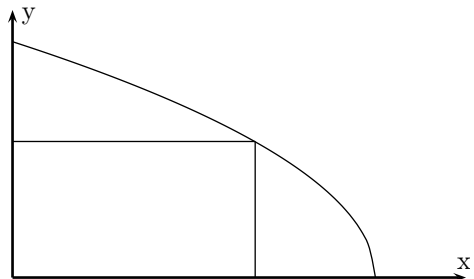
(c) Find the x -coordinates of all the inflection points for $f(x)$. (2 pts)

The inflection points are : _____

(e) Sketch the graph of $f(x)$ (you may use the fact that $\lim_{x \rightarrow -\infty} f(x) = 0$) (3 pts)



5. A rectangle is contained between the x -axis, y -axis and the curve $y = \sqrt{12 - 2x}$ as shown in the graph. What is the largest possible area of the rectangle? (10 pts)



The largest possible area is: _____

6. Gravel is being dumped from a conveyor belt at a rate of $\rho = 30 \text{ ft}^3/\text{min}$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high? (**10 pts**)

(Recall that the volume of a cone of height h and base radius r is $\frac{\pi}{3}r^2h$.)

7. Consider the parametric curve $x = 1/(t + 1)^{1/2}$, $y = t/(t + 1)$, $t > -1$

(a) Find dy/dx in terms of t . **(3 pts)**

Answer: $dy/dx =$ _____

(b) Use the value of dy/dx found in (a) to explain whether the curve has a vertical tangent. **(3 pts)**

(c) Eliminate the parameter t to find a Cartesian equation of the curve. **(4 pts)**

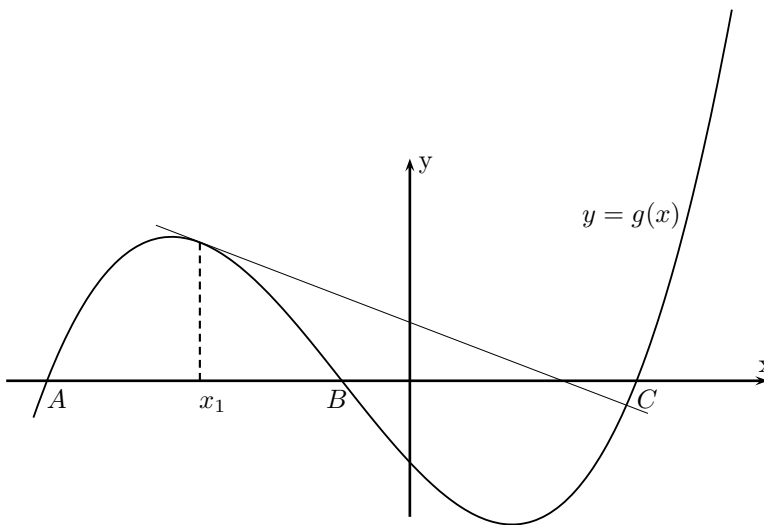
The equation is: _____

8. (a) Use Newton's method to determine the formula for the $(n + 1)$ approximation x_{n+1} of the real root of $x^3 + x = 1$. Compute the second and third approximations, x_2 and x_3 , if the first approximation is $x_1 = 0$. (5 pts)

$x_2 =$ _____

$x_3 =$ _____

(b) (5 pts)



The graph of $y = g(x)$ and its tangent line at $(x_1, g(x_1))$ are shown above. The three zeros of g are A, B , and C . Newton's method is used to approximate one of these zeros and the initial guess is x_1 , as shown in the figure. The next two approximations are x_2 and x_3 .

(i) Add to the figure above to show how x_2 and x_3 are determined. Label x_2 and x_3 on the figure above.

(ii) To which of the zeros, A, B , or C , will the initial approximation of x_1 lead? _____

9. Let $f(x) = \frac{12}{(4+x)^2}$.

- (a) Use the **midpoint rule** with a subdivision of the interval $[-3, 3]$ into three equal parts to **approximate**

$$\int_{-3}^3 f(x) dx \approx \underline{\hspace{2cm}}.$$

Show your work in a clear, well-organized, detailed way. Failure to do so will cost points. Give your answer as a fraction in the blank provided above. Do not round.

- (b) Use the Fundamental Theorem of Calculus to give the **exact** value for the same integral. Show your work.

$$\int_{-3}^3 f(x) dx = \underline{\hspace{2cm}}.$$

10. Compute and simplify if possible

(a) (3 pts) $\int_1^4 \frac{u^3 + u^2 + 1}{u^3} du =$ _____

(b) (2 pts) $\int \frac{1 + \ln t}{t} dt =$ _____

(c) **(2 pts)** $\int e^y \sin(e^y - 1) dy =$ _____

(d) **(3 pts)** $\int_0^{1/2} \frac{1}{1 + 4t^2} dt =$ _____