Review for Math 1341 **F2009**

This course introduces the basics of the calculus. Historically, the calculus is developed from the limit concept. You should know the definitions of the derivative and integral as limit of average rate of change and limit of Riemann sums respectively.

In particular, the ***definition*** of derivative of function $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Know what this means. You must be able to calculate the derivative from definition for simple functions such as 2nd order polynomial, $1/(ax+b)$, $\sqrt{x}$, etc. Part of the test here is on the algebraic skill needed to simplify the limit expression.

Be able to calculate derivatives using shortcuts. You should memorize the shortcuts for derivatives of polynomial, exponential, logarithm, trigonometric and inverse trigonometric functions. (A table of these shortcuts is listed on the next page). Also be able to calculate derivatives using product rule, quotient rule, chain rule. You are also expected to carry out the differentiation rules abstractly such as $\{x\sin[g(x)]\}' = (1)\sin[g(x)] + (x)\cos[g(x)]g'(x)$.

Know the interpretation of the derivative as rate of change, tangent slope, velocity, etc. Particularly, know how to find the tangent line equation using derivative: $y = y_0 + m(x - x_0)$ where the slope $m = f'(x_0)$ and $y_0 = f(x_0)$.

Know the geometric information contained by the derivative and 2nd derivative: $f'(x)$ is the slope of tangent on the graph, its sign tells if the function is increasing or decreasing, $f''(x)$ tells the concavity of the function. Use this to find the maximum and minimum values of the function. Be able to apply this to a word problem: set up the quantity to be optimized as a one variable function, then use the derivatives to figure out the global maximum and minimum.

You should also know how to deal with the parametric equations $x = f(t), \quad y = g(t)$: plot the curve use your graphing calculator, cancel the parameter and convert to Cartesian equation. Know the formula $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and its geometric meaning as the tangent slope. This allows you to find the tangent line equations and where the curve has horizontal and vertical tangents.

Know the techniques of implicit differentiation, logarithm differentiation and related rates. Know what asymptotes are.

You need to be able to set up the Newton's method formula for an equation $f(x)=0$, and carry out the approximation for a few iterations.

For integrals, you need to know the interpretation of area under curve as a definite integral. And the corresponding approximation by Riemann sum. Be able to calculate the Riemann sum for a small number of terms.

The fundamental Theorem of Calculus relates the integral to (anti-)derivative. Know what it means. This allows the calculation of the integral (via anti-differentiation). Know the difference between definite integral and indefinite integral. Be able to calculate both with shortcuts. Be able to use substitution rule.

Table of derivatives shortcuts

1. $(x^r)' = rx^{r-1}$ **2.** $(e^x)' = e^x$, $(a^x)' = (\ln a)a^x$ **3.** $(\ln x)' = \frac{1}{x}$, $(\log_a x)' = \frac{1}{(\ln a)x}$

4. $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$

5. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$, $(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$, $(\arctan x)' = \frac{1}{1+x^2}$

Reverse these, you get the integral shortcuts to memorize,

1. $\int x^r dx = \frac{1}{r+1}x^{r+1} + C$, $r \neq 1$; **2.** $\int \frac{1}{x} dx = \ln x + C$; **3.** $\int e^x dx = e^x + C$;

4. $\int a^x dx = \frac{1}{\ln a} a^x + C$; **5.** $\int \sin x dx = -\cos x + C$; **6.** $\int \cos x dx = \sin x + C$;

7. $\int \sec^2 x dx = \tan x + C$; **8.** $\int \frac{1}{1+x^2} dx = \arctan(x) + C$; **9.** $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$

The final exam is cumulative, testing every topics covered from the first day of class. You are expected to know the algebra used: convert negative and fractional powers, factor a polynomial and find roots for quadratic equation $ax^2 + bx + c = 0$, etc.

Some practice final exam problems are listed on the next two pages. Final exams from the previous two years are also posted on
http://www.math.neu.edu/~ding/math1341/home.htm
Solutions to those final exams will be posted there soon.

Practice final problems:

1. Find the derivative $f'(x)$ using the <u>definition</u> of the derivative. You must show all steps of your limit calculation.

(a) $f(x)=5x^2-3x+1$ **(b)** $f(x)=\frac{2x}{5x-2}$

2. Find the following derivatives.

(a) $\frac{d}{dx}e^{\tan(\sin x)}$ **(b)** $\frac{d}{dx}x\arctan(\sin\sqrt{x})$ **(c)** $\frac{d}{dx}(x\cos x-e^{x^3-3e})$ **(d)** $\frac{d}{dx}(\frac{\sqrt{x}}{5+\cos x})$

(e) $\frac{d}{dx}(\frac{2^{\sin x}}{\ln x})$ **(f)** $\frac{d}{dx}(\ln e^{ax}+\ln b)$ a, b are constants

3. Find $\frac{dy}{dx}$ by logarithmic differentiation for:

(a) $y=\frac{x(x-10)^8e^x}{\sqrt{x-6}(x+14)^2}$ **(b)** $y=x^{x+\ln x}$.

4. Let $g(x)=e^{1+xf(x)}$. Find $g'(1)$ if $f(1)=-1$ and $f'(1)=2$. Find the equation of the tangent line to $y=g(x)$ at the point (1,1).

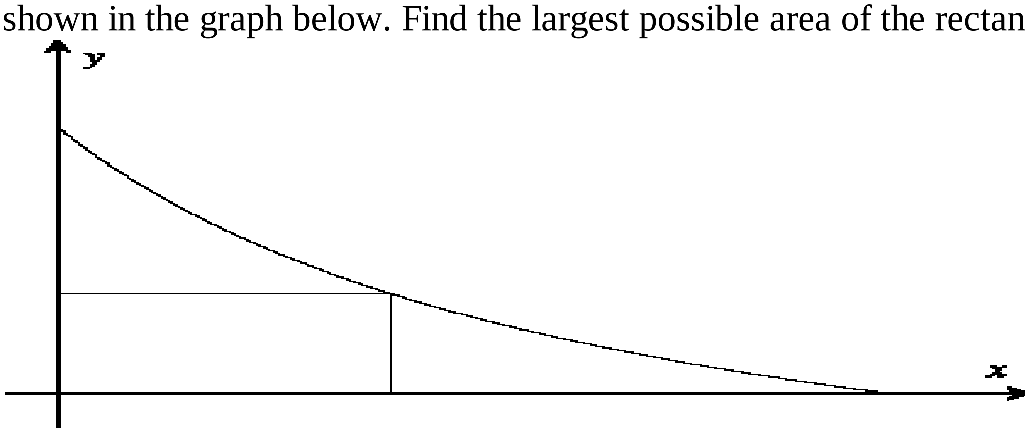
5. Find $\frac{dy}{dx}$ for $x^4y^2+2y=3x$. Find the equation of the tangent line to this curve at the point (1,1).

6. For the parametric curve $x=\arcsin t,\quad y=t+2$. Find the slope of the tangent line when t=0. Find the equation of the tangent line when t=0. Does the curve have horizontal tangent? Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve.

7. Cyclist A and B starts out at 1:00 PM from a tower and going away at two different directions. Cyclist A rides south at the constant speed of 20 miles per hour. Cyclist B rides west at the constant speed of 15 miles per hour. At 3:00 PM, what is the rate at which the distance between the cyclists is increasing?

8. Set up Newton's method formula applied to the equation $x^3-5=0$. Find the third approximations x_3 for the root if the first approximation is $x_1=2$.

9. A rectangle is contained between the x-axis, the y-axis and the curve $y=\frac{3-x}{x+2}$ as shown in the graph below. Find the largest possible area of the rectangle.



10. *derivative* $f'(x)=\frac{x}{x^2+1}$

(a) Find the intervals of increasing for the function $f(x)$.

(b) Find the interval on which the function $f(x)$ concave upwards.

(c) Find the x values of local maxima, local minima and inflection points.

11. Sketch the graph of a function with

derivative $f'(x)<0$ on $(-\infty,0)$ and $(10,\infty)$; $f'(x)>0$ on $(0,10)$;

2nd derivative $f''(x)<0$ on $(-\infty,-10)$ and $(5,\infty)$; $f''(x)>0$ on $(-10,5)$.

and $\lim_{x\to-\infty} f(x)=0$

12. Calculate the Riemann sum for $\int_0^{\pi}\sin x\,dx$ with n=2, using **(a)** right endpoints; **(b)** left endpoints; **(c)** midpoints. Which of the three answers above is closest to the extra value of the integral?

13. Find $f(x)$ for

(a) $f'(x)=3^x+2\sin x$ **(b)** $f'(x)=\frac{1}{\cos^2 x}-\sqrt{x}$ **(c)** $f'(x)=\frac{\ln x}{x}-7x^2+2$

14. Find

(a) $\int_0^{\pi/3}\cos(2x)dx$ **(b)** $\int(x^e+e^x)dx$