

Scores by problem:

1	2	3	4	5	6	7	8	9	10	Total
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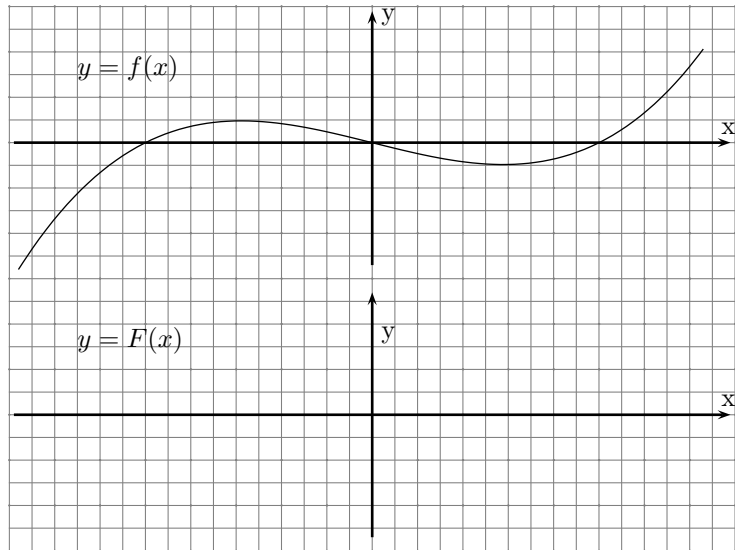
Instructor: \_\_\_\_\_ Name: \_\_\_\_\_

**NORTHEASTERN UNIVERSITY**  
**MTH U241 (Calculus I)      Final Exam      Fall 2007**

**SHOW YOUR WORK.** There is **NO credit for the answer without the work shown.** If there is not enough room to show your work, use the back of the preceding page.

1. (a) For the function  $f(x) = \frac{1}{x-1}$  write down the value of the derivative  $f'(x)$  at  $x = 5$  as the limit of an algebraic expression, and compute the limit by simplifying this algebraic expression. **(5 pts)**

(b) The graph of function  $f$  is given bellow. Use it to sketch the graph of its antiderivative  $F$  satisfying  $F(0) = 0$ . On the graph of  $F$  indicate the points where the graph of  $F$  is concave upwards. **(5 pts)**



**2.** Calculate the following derivatives. If possible, simplify your answer.

(a) **(2 pts)**  $\frac{d}{dx} (x \ln x - x + 2)$

(b) **(2 pts)**  $\frac{d}{dr} \frac{3r}{5r + 2}$

(c) **(3 pts)**  $\frac{d}{dt} [e^{4t} + \tan(3t)]$

(d) **(3 pts)**  $\frac{d}{dx} [(1 + x^2)^x]$

**3.** Consider the curve defined by:  $y = xe^y + 1$ .

(a) Find the expression for  $dy/dx$  as a function of  $x$  and  $y$ . **(3 pts)**

(b) Find the equation of the tangent line to the curve at  $(-1, 0)$ . **(4 pts)**

(c) Use the linear approximation of  $y$  at  $x = -1$  to estimate the  $y$ -coordinate of the point on the curve with  $x$ -coordinate being  $-1.02$ . **(3 pts)**

4. For the function  $f(x) = e^{-x^2}$  find the following. (You should use calculus and show your work; you can use your graphing calculator to check your answer.)

(a) Intervals of increase or decrease. **(3 pts)**

(b) Local maximum and minimum values. **(3 pts)**

(c) Intervals of concavity and inflection points. **(4 pts)**

5. A closed box of volume 800 cubic inches is required to have a base whose length is 4 times its width. Find the dimensions of the box that has the least surface area. (10 pts)

6. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 2 feet per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall ? **(10 pts)**

7. Consider the parametric curve  $x = 4t^2 - 2t$ ,  $y = 2t + 1$ .

(a) Eliminate the parameter  $t$  and find the equation in  $x, y$  that defines the curve. **(4 pts)**

(b) Find the value of  $t$  for which the curve has a vertical tangent. **(3 pts)**

(c) Sketch the curve and the vertical tangent. **(3 pts)**

8. (a) Set up Newton's method for finding a root of the equation  $e^x = 5 \cos x$ : indicate to which function you apply Newton's method, and what is the formula for  $x_{n+1}$  in terms of  $x_n$ . **(5 pts)**

(b) Using your calculator, find a root of the equation  $e^x = 5 \cos x$  correct to 8 decimal places using the initial approximation  $x_1 = 1$ . Show the numbers  $x_2, x_3, \dots$ . **(5 pts)**

9. (a) Using the midpoint rule, and four subintervals, approximate the area under the function  $y = x^3 - 5x^2 + 30$  from  $x = -2$  to  $x = 6$ . **(7 pts)**

(b) Using the Fundamental Theorem of Calculus find this area exactly. **(3 pts)**

10. Compute and simplify if possible

(a) **(3 pts)**  $\int \left( 3\sqrt{x} + \frac{2}{x^2} - \frac{1}{x} \right) dx$

(b) **(3 pts)**  $\int_1^4 \frac{u^3 + u}{u^2} du$

(c) **(2 pts)**  $\int e^x \cos(e^x + 1) dx$

(d) **(2 pts)**  $\frac{d}{dx} \int_1^{e^x} \ln t \, dt$