

Northeastern University

Department of Mathematics

Practice Final Exam

MTH U121

Fall 2006

1	2	3	4	5	6	7	8	9	10

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Name: _____

Instructor: _____

1a. Solve for x (Hint: simplify first) $\frac{x^{\frac{11}{3}}}{\sqrt[3]{x^4}} \cdot (x^{-10})^{\frac{1}{15}} = 3125$

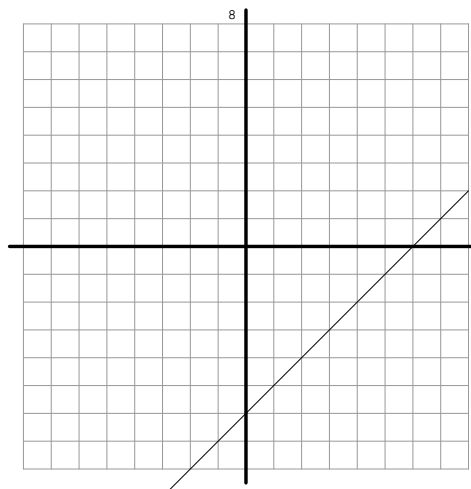
1b. Simplify (Hint: factor first) $\frac{x^2 + 2x - 3}{x^2 + 7x - 8} \cdot \frac{x^2 + 12x + 32}{x^2 + 13x + 36} =$

1c. Solve for x : $\frac{7x}{7x - 4} - \frac{5}{7 - 5x} = 1$

2a. A librarian is shipping boxes of books. The cost, C , of shipping is directly proportional to the weight (in pounds), W , of the books. The company advertises that 18,000 pounds of books can be shipped for \$810.

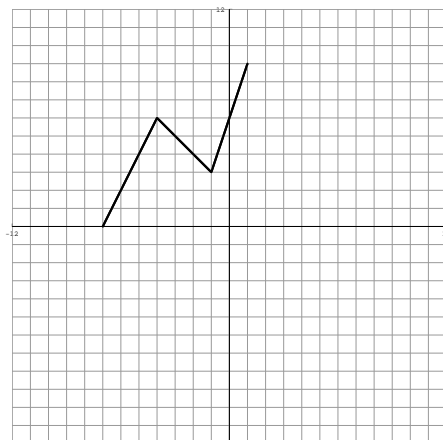
- i. How much does it cost to ship 51,000 pounds of books?
- ii. How many pounds of books can be shipped for \$2,430?

2b. Here is the graph of a linear function. Find the function in the form $y = mx + b$.



3. The sketches below show the graph of a piecewise linear function $y = f(x)$.

- i) What is the value of $f(-1)$ =
- ii) Find the average rate of change from $x = -7$ to $x = 1$.
- iii) Add to the graph the sketch of $y = f(x - 4) - 3$
- iv) Add to the graph the sketch of $y = \frac{1}{3} \cdot f(x)$

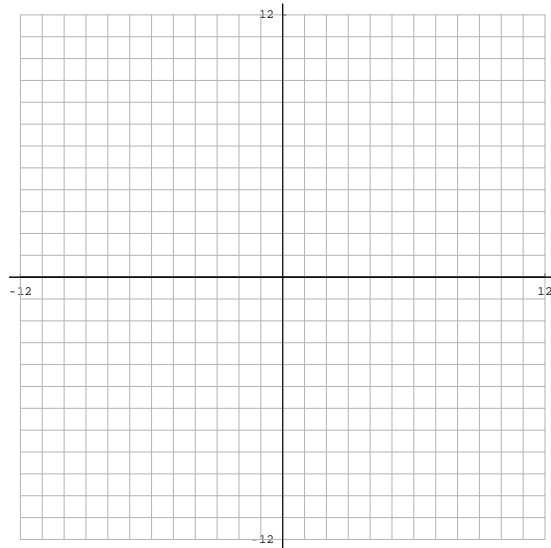


4a. Sketch the graph of $y = \frac{x^2}{2} + 2x - 2$. Indicate the points found below on the graph.

b. Find the y -intercept.

c. Find the vertex.

d. Find the x -intercepts (if any).



5. A project in a business class involves building and selling picture frames. The students are selling 160 a week at \$12 each. The materials for each cost \$10 a piece. A carefully done survey indicates that for every dollar increase in price the number of sales per week will decrease by four.

a. What is their current weekly profit?

b. Find a (quadratic) formula (in the form $ax^2 + bx + c$) for the profit as a function of the price.

c. Find that price that maximizes the profit and determine the maximum profit.

6a. Do the following calculations and put the answer in the form $a + bi$ (and put fractions into reduced form.)

i. $(9 + 5i) \cdot (2 + i) =$ ii. $\frac{4 + 5i}{2 - i} =$ iii. $\frac{1}{4 + 7i} + \frac{1}{4 - 7i} =$

6b. Find all (real and complex) solutions of $x^3 - 10x^2 + 61x = 0$.

6c. Find a polynomial with integer coefficients (in standard polynomial form $a_nx^n + \dots + a_1x + a_0$) such that P has degree three, and zeros 12 and $6 - 5i$

7. A biologist is developing a new strain of fruit flies. Careful examination reveals that 1,500 fruit flies grows to 3,250 in nine weeks. Assuming that the growth rate follows the exponential law $A(t) = A_0e^{kt}$, where t denotes time measured in weeks, answer the following.

a. Find the value of the constant k .

b. Starting from 1500 fruit flies, how many will there be in twelve weeks?

c. How many weeks does it take for 1,500 fruit flies to grow to 6,000?

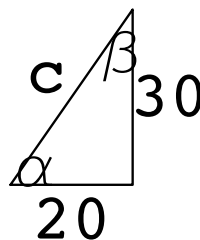
8a. A research ship from Woods Hole is tracking an errant iceberg in the Atlantic ocean. At 6:50 PM on July 18 the iceberg was at latitude 35 degrees, 25 minutes. The ship was due south of it at latitude 30 degrees, 45 minutes. Using $r = 3960$ miles for the radius of the Earth determine how far the ship is from the iceberg.

8b. Solve the triangle by finding the values of

i. $c =$ _____

ii. (in degrees) $\alpha =$ _____

iii. (in degrees) $\beta =$ _____.



8ci. From the graph, **find** $y = A \cdot \sin(\omega \cdot t - \phi) + S$

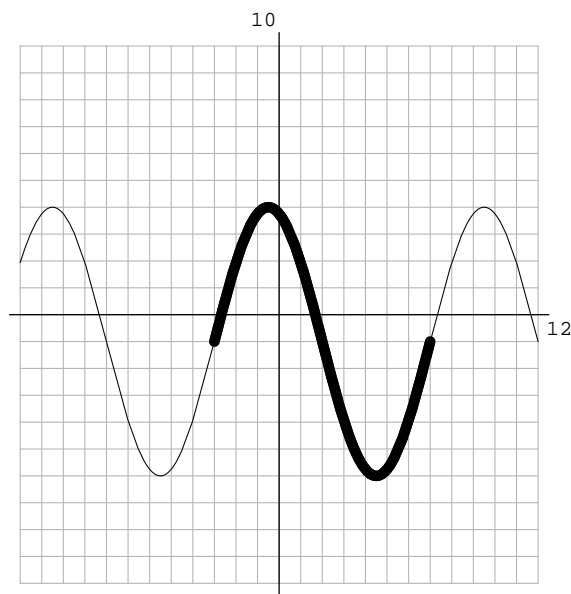
ii. What is the amplitude? _____

iii. What is the vertical shift? _____

iv. What is the wavelength? _____.

v. What is the phase shift (starting x-value)? _____

i. $y =$ _____



9a. Solve the following system of equations. (Use any method, but show your work.)

$$\begin{cases} x + 2y = -15 \\ x^2 + y = 3 \end{cases}$$

9b. A laboratory is supplied with two containers of salt solutions. The first is a 60 percent solution and the second is a 10 percent solution. For an experiment you need 1,000 milliliters of a 50 percent salt solution. How much of each should you mix in order to get the required salt solution?

10. Solve the following system of linear equations for x, y and z . (*You may use your calculator to check your answer-but you still must show the computational work to receive credit.*)

$$\begin{aligned} 2x + 3y + z &= -8 \\ 3x + 5y - z &= -11 \\ 5x + 6y + 2z &= -21 \end{aligned}$$

Some Formulas you may need

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponential/Logarithmic Relation: $y = e^x \iff \ln(y) = x$, and $y = 10^x \iff \text{Log}(y) = x$.

Exponential Rules: $\frac{x^a x^b}{x^c} = x^{a+b-c}$, and $(x^a)^b = x^{ab}$.

Logarithmic Rules: $\ln\left(\frac{ab}{c}\right) = \ln(a) + \ln(b) - \ln(c)$,
 $\ln(b^u) = u \cdot \ln(b)$ and $\ln(a \cdot b^u) = \ln(a) + u \cdot \ln(b)$.

Exponential Growth/Decay: $A(t) = A_0 e^{kt}$. Compound Interest: $A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}$

Degree/Radians: $360^\circ = 2\pi$ radians. Deg/Min/Sec: $1^\circ = 60'$, $1' = 60''$.

Arclength: $s = r \cdot \theta$, (θ must be in radians).

Sinusoidal Function: $y = A \cdot \text{Sin}(\omega \cdot x - \phi) + B$, Period = $T = \frac{2\pi}{\omega}$, Phase Shift = $\frac{\phi}{\omega}$.

Trigonometric Definitions: $\text{Sin}(\theta) = \frac{\text{opp}}{\text{hyp}}$, $\text{Cos}(\theta) = \frac{\text{adj}}{\text{hyp}}$, $\text{Tan}(\theta) = \frac{\text{opp}}{\text{adj}}$.