

NORTHEASTERN UNIVERSITY

Department of Mathematics

MTH U121

**FINAL EXAMINATION**

Spring 2004

1	2	3	4	5	6	7	8	9	10

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Name: \_\_\_\_\_

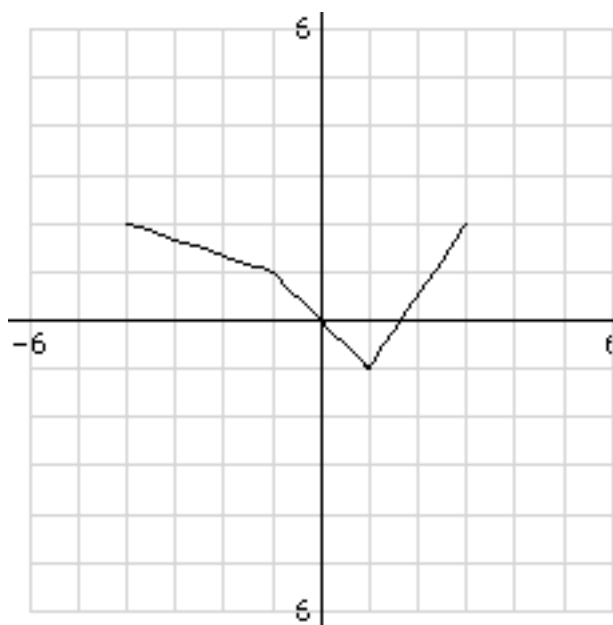
Instructor: \_\_\_\_\_

1. A student spends twelve hours, from 8:00 PM to 8:00 AM, as a temporary employee at a large company with many different departments. The hourly pay rate varies and depends upon the department in which the student works at any given moment. The first three hours the student works in Department A and earns \$28.50 and then takes a half hour break. After the break, the student returns to Department A for two and a half hours and then switches to Department B. Department B pays eleven dollars an hour and the student is there for a total of three and a half hours. After a second thirty minute break the student goes to Department C. Department C pays the same rate as Department A, and the student finishes the work shift there.
  - a) What is the hourly pay rate in Department A?
  - b) How long did the student work in Department C?
  - c) What is the total amount the student earned?
  - d) Draw a graph of the student's earnings as a function of time. Clearly indicate the units and scales used on your graph.

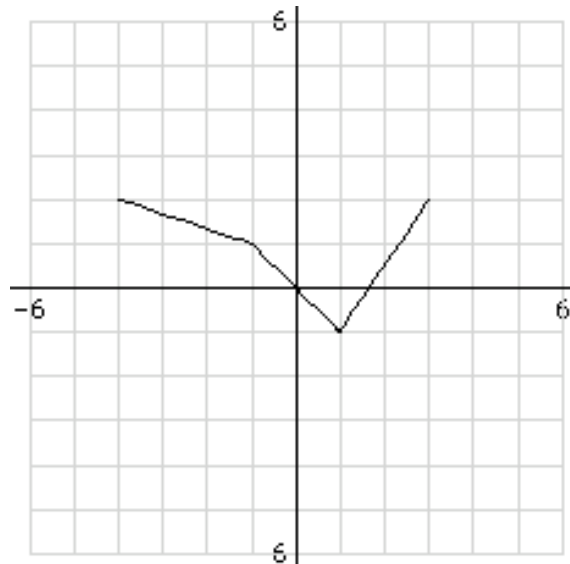
2. The sketches below show the graph of a piecewise linear function  $y = f(x)$ .

a) What is  $f(-4) =$

b) Add to the sketch the graph of  $y = f(x-1) - 3$ .



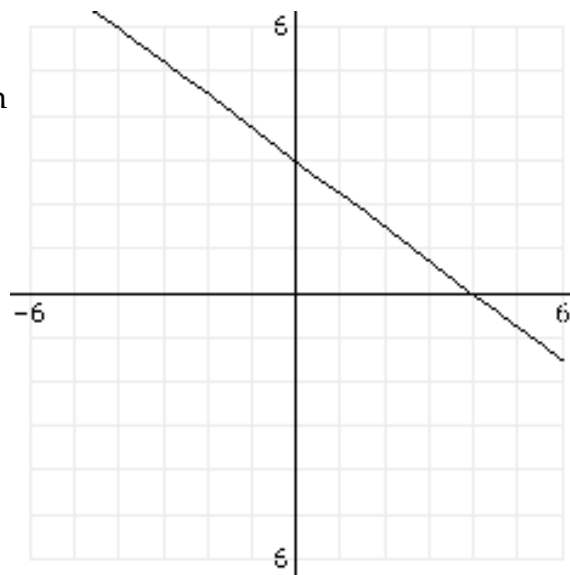
- c) Add to the sketch the graph of  $y = 2 \cdot f(x)$ .



3a. The cost  $C$  of buying gasoline at a neighborhood station varies directly with the number of gallons purchased. If twenty-two gallons cost \$41.58 how much does ten gallons cost?

3b. How many gallons can be purchased for ten dollars?

- 3d. Here is the graph of a linear function.  
Find the equation of the function in the form  $y = mx + b$ .

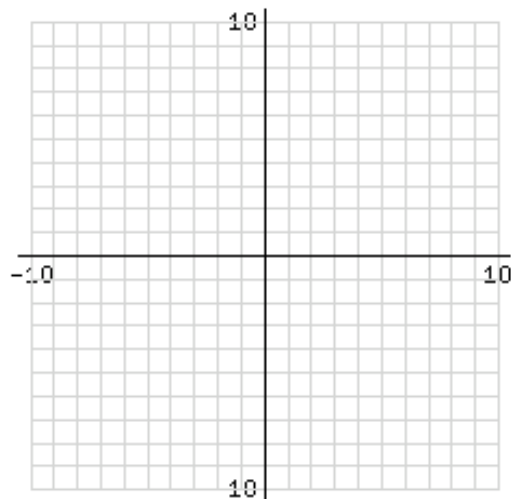


4a. Sketch the graph of  $y = \frac{1}{4}x^2 - 2x - 3$ .

b) What is the vertex? Give both  $x$  and  $y$  coordinates

c) What is the  $y$ -intercept?

d) What are the  $x$ -intercepts if any. If there are none, state so.



5. A cartel of mushroom harvesters has determined that the price  $p$  and quantity  $x$  (in grams) for french truffles obeys the demand function

$$p = -x/8 + 48, \quad 0 \leq x \leq 384.$$

- What is the revenue  $R$  if 16 grams of truffles are sold?
- Express the revenue  $R$  as a function of  $x$ .
- What quantity  $x$  maximizes the revenue? And what is the maximum revenue?
- What price should the company charge in order to maximize the revenue?

6a Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the real and complex roots of the polynomial.

$$A(x) = (x^2 - 8x + 7)(81 - x^4)$$

b. Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the roots of the polynomial (both real and complex).

$$B(x) = (x^4 - 8x^2 + 7)(6 - x^2)$$

c. Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the roots of the polynomial (both real and complex).

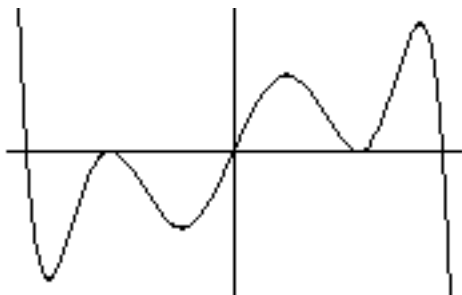
$$C(x) = (x^2 - 9)^2(25x - x^3)$$

d. The graphs of two of the following functions are drawn below:

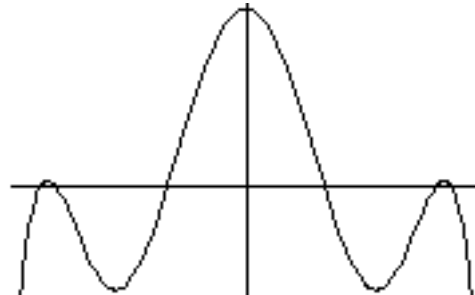
$$A(x) = (x^2 - 8x + 7)(81 - x^4);$$

$$B(x) = (x^4 - 8x^2 + 7)(6 - x^2); \quad C(x) = (x^2 - 9)^2(25x - x^3)$$

Under each graph, identify the function by writing its name (e.g.  $A(x)$ ).



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7. A newly discovered colony of bacteria is observed to grow from 1500 to 2100 in four hours. Assume the bacteria grows exponential according to  $A(t) = A_0 e^{kt}$ .

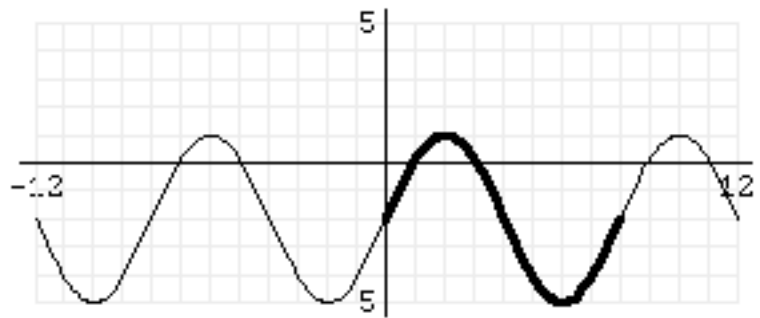
- a) Find the value of the growth constant k.
- b) How much bacteria will there be in five hours (starting from when there was 1500 bacteria)?
- c) How long will it take the bacteria to grow from the initial population of 1500 to 2500?

8. Assuming the Earth is a sphere of radius 6378 kilometers, what is the difference in latitudes of two cities if one is 2,126 kilometers due north of the other? Give your answer in radians and then convert to degrees, minutes and seconds

9a. At 5:20 AM this morning, April 23, 2004 there was a high tide at Castle Island in Boston Harbor. According to XTide the Harmonic tide clock and tide predictor, the low tide should occur at 11:27 AM. Using  $t = 0$  to denote the beginning of the day, these times are respectively  $t = 5.3333$  hours and  $t = 11.45$  hours. If the heights of the high and low tides are 7.84 feet and 2.49 feet, determine a sinusoidal function of the form  $y = A \sin(\omega t - f) + B$  from this data. (You may assume that the time between high tides is 12.5 hours).

b) What does your formula predict the height of the water will be at noon today?

9c) Find an equation for the function given by the following graph



10 Solve the system of linear equations for x, y and z. (You may use your calculator to check your answer - but you still must show the computational work to solve the problem.)

$$\begin{aligned} x + y + z &= 1 \\ 2x - 3y + 6z &= 1 \\ 3x - 3y - 3z &= 0 \end{aligned}$$