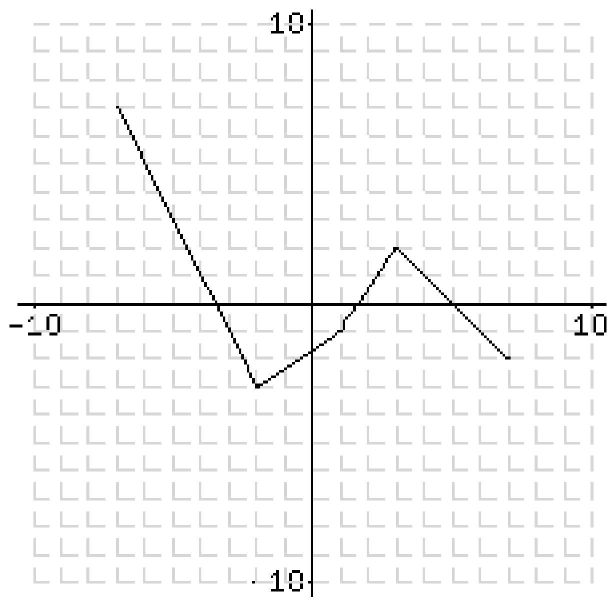
Name ______________________

Practice Final Exam

1. Word/Graph Connection of functions: Section 2.2 37-42.
   A skydiver jumps from an airplane and falls free for the first six seconds)where distance fallen is proportional to the square of the time fallen). Then after having falling 576 feet, the parachute opens and the skydiver descends to the ground at a constant speed of 14 feet per second. The ground is reached 100 seconds after the parachute opens. (i) how high was the airplane when the skydiver jumped? (ii) Make a sketch of the skydiver's height above the gorund as a function of time during the course of the descent. Include in the sketch the coordinates of the points corresponding to when the skydiver first jumps, when the parachute opens, and when the skydiver reaches the ground.

2. Transformations of graphs: Sec 2.6 pg 128 65-68, pg 127 7-17.

   Here is the graph of a function $y = f(x)$.



a) What is $f(-2)$?

b) Sketch the graph of $y = 5 - f(x-2)$

c) Sketch the graph of $y = 5f(x)$

d) Sketch the graph of $y = f(5x)$

3. Linear word problem Sec 2.4 pg 103 39-48.

a) A baker specializes in wedding cakes. It costs the baker \$185 for ingredients and labor for each cake, and there are fixed weekly costs of \$1,250 for rent, water, gas and electricity. Write a linear function that expresses the cost of producing x cakes per week. Graph the linear function. What is the cost of producing twenty-five cakes in a week. Because of cash flow considerations, the baker must keep next week's cost below \$10,000. How many cakes should the baker produce next week to meet this restriction while still producing the maximum number of cakes?

b) The monthly payment p on a mortgage varies directly with the amount borrowed B. If the monthly payment on a twenty-five year mortgage \$7.23 for every \$1,000 borrowed, find a linear function that relates the monthly payment p to the amount borrowed B for a mortgage with the same terms. Find the monthly payment when the amount borrowed is \$300,000. If a borrower wants to limit the monthly payments to \$1,500 a month, what is the maximum that can be borrowed?

4. Graphs of Quadratic Functions: Sec 3.1 pg 164 35-58.

a) Sketch the graph $y = 5x^2 + 2x - 3$. Find the vertex, y-intercept and x-intercepts if any.

b) Sketch the graph $y = 5x^2 - 3x - 3$. Find the vertex, y-intercept and x-intercepts if any.

Practice Final Exam

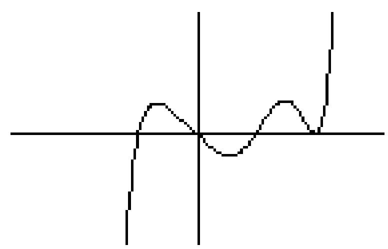
5. Quadratic Word Problem/Optimization: Sec 3.1 pg 165 71-76.

a. The marginal cost of a product can be thought of as the cost of producing one additional unit of output. A company produces a product and finds that the marginal cost of producing the x^{th} item is $C(x) = x^2 - 70x + 1{,}750$. What is the marginal cost to product the tenth item? How many items should be produced in order to minimize the marginal cost?

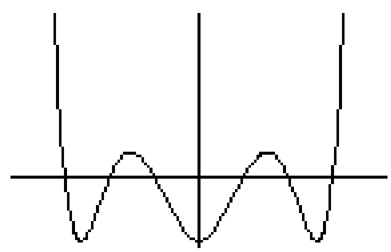
b. The price p and the quantity x sold of a certain product obey the demand equation $x = -35p + 1750,\ 0 = p = 50$. Express the revenue R as a function of the quantity x. Express the revenue R as a function of the price p. What is the revenue if twenty-five units are sold? What is the revenue if the price is twenty-five dollars? What quantity maximizes the revenue and what is the maximum revenue? What price maximizes the revenue and what is the maximum revenue?

6. Analyzing Polynomials - Roots, Properties , Factoring: Sec 3.2, 3.6, 3.7, A.5, A.6 pg 998 81-89, pg 1008 53-72, pg 237 7-30, pg 231 ????

a) Here is the graph of a polynomial of degree seven. All of its roots are of multiplicity one or two. How many complex roots does it have? How many roots of multiplicity two does it have? Is 0 a root of the polynomial?



b) Here is the graph of a polynomial with only real roots, and all the real roots are of multiplicity one. Is the polynomial of even degree or odd degree? What is the degree of the polynomial? If you are told that $x = -4$ is a root of the polynomial, do you know if $x = 4$ is also a root of the polynomial? Explain your reasoning.

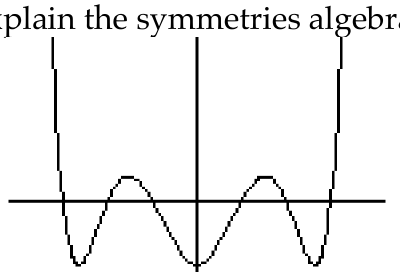


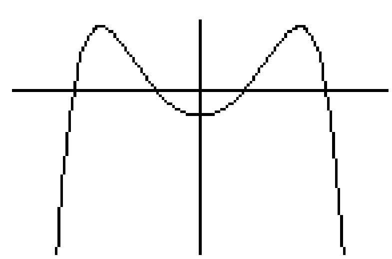
c) i) Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the roots of the polynomial. $f_1 = (x^2+6x-7)(256-x^4)$.

ii) Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the roots of the polynomial. $f_2 = (x^2-4)^2(9x-x^3)$.

iii) Factor the polynomial into linear and quadratic terms with real coefficients. Then find all the roots of the polynomial. $f_3 = (x^4+6x^2-7)(8-x^2)$.

iv) Match the three functions f_1 f_2 f_3 with the graphs. What symmetries do the graphs have? Explain the symmetries algebraically.





Practice Final Exam

7 Compound Interest, Exponential Growth and Decay: Section 4.7, Section 4.8

a. What rate of interest, compounded monthly, allows $5,000 to grow to $20,000 in twenty years?

b) How much must you invest today in order to have $20,000 in twenty years if the interest rate is seven and a half percent per year compounded semiannually?

c) The bank is advertising an investiment which pays simple interest (compounded annually) which allows $1,000 to grow to $5,000 in twenty years. If you only want the money to grow to $2,500, how long must you wait?

d) A colony of bacteria can grow from 2,500 to 10,000 in seven hours. Starting from when it has 2,500 bacteria how much will the colony have in ten hours?

e) How long will it take a colony of bacteria to grow from 5,000 to 20,000 if it can grow from 5,000 to 7,500 in three hours?

f) If a colony of bacteria can double in size in three hours, how long does it take it to triple in size?

8. Trigonometry - Angles and their Measures, Arclength: Sec 5.1 pg 368 91, 101, 102

a) City A is due north of City B. The latitude of A is $62°28'31''$, and the latitude of B is $31°62'\ 28''$ Using 3,950 miles for the radius of the earth, find the distance between the two cites.

b) City C is 550 miles due south of city D. Using a radius of 3,960 miles for the Earth, find the difference in latitudes of the two cites in degrees, minutes and seconds.

c) Writing from a long extinct, but very scientific civilization has been recently discovered in Iceland. From the writing we have discovered that they had two primary cities, one of which was $\pi/6$ radians north of the other. The distance between these two cites is recorded as 131 raakiins. Unfortunately there is no indication of how long a raakiin was in miles. What was the radius of the Earth in raakiins? How many miles are in a raakiin?

9. Trigonometry - Graphs and Sinusoidal Curve Fitting Sec 5.4, Sec 5.6 pg 416 47-88, pg 435 21-30.

a) For any given city the hours of sunlight in a day cycles annually with a period of 365 days. If $t = 0$ represents January 1, then the longest day is the summer solstice which in 2001 was on June 21 ($t = 172$) and the shortest day is the winter solstice which in 2001 was on December 21 ($t = 355$). A small town in Canada had 17.23 hours of sunlight on the summer solstice and 6.56 hours on the winter solstice. Find a sinusoidal function of the form $A \cdot \mathrm{Sin}(\omega t - \phi) + B$ that fits the data. Using this model estimate the amount of sunlight on April 1 ($t = 91$).

b) In this same city the length of time between high tides happens to be 12.25 hours. On the summer solstice the first high tide was 2:23 AM in the morning and the first low tide was at 8:52 AM. When was the second high tide that day? Using $t = 0$ to represent the beginning of the day, the first high tide is represented by $t = 2.38333$ and the low tide by $t = 8.86666$. If the height of the high tide was 9.8 feet and the low tide was 3.6 feet, find a sinusoidal function of the form $A \cdot \mathrm{Sin}(\omega t - \phi) + B$ that fits the given data. Use your model to predict the height of the water at 1:30 PM.

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Practice Final Exam

10. Systems of Linear Equations and Matrices: Sec 10.1, Sec 10.2, pg 739 51-54, pg 755 37-40, 43-50.

a) Solve the system for x,y,z

$$\begin{aligned} x - y - z &= -3 \\ 2x - y + z &= -3 \\ 3x + y - 3z &= 7 \end{aligned}$$

b) Solve the system for x,y,z

$$\begin{aligned} 2x + y + z &= 0 \\ 3x - 2y + z &= -1 \\ 5x + y + 3z &= -1 \end{aligned}$$

FORMULAs

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exponential/Logarithmic Relation: $y = e^x \Leftrightarrow \ln(y) = x$; $y = 10^x \Leftrightarrow \log(y) = x$

Exponential Rules: $\dfrac{x^a x^b}{x^c} = x^{a+b-c}$, $\left(x^a\right)^b = x^{ab}$

Logarithmic Rules: $\ln\left(\dfrac{a \cdot b}{c}\right) = \ln(a) + \ln(b) - \ln(c)$; $\ln\left(b^u\right) = u \cdot \ln(b)$; $\ln\left(a \cdot b^u\right) = \ln(a) + u \cdot \ln(b)$

Exponential Growth/Decay: $A(t) = A_0 e^{kt}$; Compound Interest/Depreciation: $A(t) = A_0\left(1 + \dfrac{r}{n}\right)^{nt}$

Degree/Radians Conversion: $360^\circ = 2\pi$ radians; Deg./Min./Sec: 1°=60'; 1' = 60''

Arclength: $S = R \cdot \theta$; Area of a Sector: $A = \dfrac{1}{2}R^2 \cdot \theta$; Circular Motion: $V = R \cdot \omega$

Sinusoidal Function: $y = A \cdot Sin(\omega \cdot x - \varphi) + B$

Amplitude $= |A|$; Vertical Shift = B; Period $T = \dfrac{2\pi}{\omega}$; Phase Shift = Starting-x $= \dfrac{\varphi}{\omega}$