Practice Final Exam, MathU242, Spring 2004

Show All Your Work. No Show, No Credit

1. a. Solve the differential equation $y' = \left(4t^3 + 4t\right)\cdot(y + 3)$.

   b. If, in addition, y(1) = 7, solve that initial value problem.

2. Find the volume of the parallelepiped determined by **a** = <3,2,0>, **b** = <4,-2,0> and **c** = <3, 1, 5>.

3. Let **u** = **2i** – **j** + **k** and **v** = **i** + **j** + 2**k**. Find the following:

   a. **u**x**v**

   b. The length of **u**

   c. The length of **v**

   d. The angle θ between **u** and **v**.

   e. A vector **w** of length 1 in the direction of **v**.

   f. Proj_w(**u**)

4. Do the following integrals diverge or not? Give an appropriate limiting argument and calculate the number if the value of the integral is a number.

   a. $\int_{-1}^{\infty} \frac{x\,dx}{1 + x^2}$

   b. $\int_{1}^{2} \frac{dx}{1 - x}$

5. a. Find the first four non zero terms of the power series expansion of f(x) = $\frac{1}{4 + x^2}$, which is centered at x = 0.

   b. Use your answer from #5.a. above to determine $f^{(6)}(0)$.

6. Suppose that Q(x) is the first three non zero terms of the Taylor's series for sin(2x), centered at x = 0. Estimate the maximum size of | Q(x) – sin(2x) | on the interval $-\frac{p}{2} < x < \frac{p}{2}$. If you use a calculator, round to three places and be sure to round up.

7. Write out a Trapezoidal rule estimate, with n = 4, for $\int_0^p e^{\sin x}\,dx$. **DO NOT EVALUATE.**

8. Write out a Simpson's rule estimate with n = 6, for $\int_{\frac{p}{4}}^{p} \ln(\sin x)\,dx$. **DO NOT EVALUATE.**

9. If A = $\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -2 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$, calculate the following, or state that it is undefined:

a. AB

b. BA

c. det(A)

d. det(B)

10. Calculate the determinant of A = $\begin{pmatrix} 23 & 57 & 479 \\ 23 & 57 & 480 \\ 2 & 3 & 481 \end{pmatrix}$.

11. Find the area of the polar region bounded by the curve r = 3 $\sqrt{\sin q}$ and the rays $q = \frac{p}{3}$ *and* $q = \frac{p}{2}$.

12. a. If (x, y) = (2, 1) give the polar coordinates of the point.

b. If (r, θ) = $\left(\sqrt{2}, \frac{3p}{4}\right)$ give he Cartesian coordinates of the point.

13. Find the volume generated when the region bounded by the curve y = $\sqrt{x}$ and the lines y = 2 and x = 0 is rotated about:

a. The x-axis

b. The line x = 4.

14. A parametric curve is given by the equations

$r(q) = (\cos q + q\sin q, \sin q - q\cos q)$. Find the arc length along the curve from $q = 0$ *to* $q = \frac{p}{2}$.

15. Use the cylindrical shell method to find the volume of the solid generated by revolving the region bounded by the curves y = 2x – 1, y = $\sqrt{x}$ and x = 0 around the y-axis.

16.An oddly shaped tank has its bottom at y = 0 and its top at y = 9 meters. The cross sectional area at height y meters is given by A(y) = 4√y square meters. The tank is full of a liquid that weighs 5 Newtons per cubic meter. (Note: You are given the WEIGHT per cubic meter, not the mass. You need not multiply by g.) How much work is required to lift all of the liquid up to the top edge?

17. Evaluate the following integrals:

a. $\int \sin^{-1} x dx$

b. $\int x^2 \tan^{-1} x dx$

18. find the area of the region bounded by the two curves y = $-x^2$ and y = x^3 for -1 = x = 0.

19. Use partial fraction s to evaluate the integral $\int \frac{(X + 23)dx}{x^2 + 4x - 21}$

20. Use the Taylor's series for sinx to find a Taylor's series for sin($7x^3$).

21. Find the sum of the series $\sum_{k=1}^{\infty} \frac{(x + 5)^k}{7^k k}$. (Your answer will involve x.)

22. Find the radius of convergence of thee power series $\sum_{k=2}^{\infty} \frac{k!\cdot(x + 7)^k}{(k + 2)!}$