Quiz 9A, answers

1) Find a unit vector which points in a direction perpindicular to the vectors $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

First, calculate the cross product of $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, which will be perpindicular to both vectors.

$$\det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 1 \\ -1 & 2 & 2 \end{pmatrix} = \mathbf{i}(-6-2) - \mathbf{j}(4+1) + \mathbf{k}(4-3) = -8\mathbf{i} - 5\mathbf{j} + \mathbf{k}$$

Then, multiply by the reciprocal of its length to get a unit vector.

$\| -8\mathbf{i} - 5\mathbf{j} + \mathbf{k} \| = \sqrt{64 + 25 + 1} = \sqrt{90} = 3\sqrt{10}$

Answer: $\dfrac{1}{3\sqrt{10}}(-8\mathbf{i} - 5\mathbf{j} + \mathbf{k})$

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2) Suppose $A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ x & 0 \\ 0 & 2 \end{pmatrix}$.

a. If AB is defined, calculate it. Otherwise, simply say that it is undefined.

$$AB = \begin{pmatrix} 2+3x & 5 \\ 4-x & 10 \end{pmatrix}$$

b. If BA is defined, calculate it. Otherwise, simply say that it is undefined.

$$BA = \begin{pmatrix} 8 & 3 & 8 \\ x & 3x & x \\ 4 & -2 & 4 \end{pmatrix}$$

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3) Suppose $A = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & x \end{pmatrix}$ and $B = \begin{pmatrix} y & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix}$.

a. Calculate $\det(A)$.

$$\det(A) = +2\det \begin{pmatrix} 1 & 1 \\ 0 & x \end{pmatrix} - 2\det \begin{pmatrix} 0 & 1 \\ 0 & x \end{pmatrix} + 2\det \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = 2x.$$

b. Calculate $\det(B)$.

$$\det(B) = +y\det \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix} - 0\det \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} + 0\det \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 2y.$$

c. Calculate $\det(AB)$.

$$\det(AB) = \det(A)\det(B) = 4xy.$$

d. Calculate $\det(BA)$.

$$\det(BA) = \det(B)\det(A) = 4xy.$$

Quiz 9B, answers

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1) Find a unit vector which points in a direction perpindicular to the vectors $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}+2\mathbf{k}$.

First, calculate the cross product of $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}+2\mathbf{k}$, which will be perpindicular to both vectors.

$$\det\begin{pmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 3 & 1\\ -1 & 2 & 2\end{pmatrix} = \mathbf{i}(6-2)-\mathbf{j}(4+1)+\mathbf{k}(4+3) = 4\mathbf{i}-5\mathbf{j}+7\mathbf{k}$$

Then, multiply by the reciprocal of its length to get a unit vector.

$\| 4\mathbf{i}-5\mathbf{j}+7\mathbf{k} \| = \sqrt{16+25+49} = \sqrt{90} = 3\sqrt{10}$

Answer: $\dfrac{1}{3\sqrt{10}}(4\mathbf{i}-5\mathbf{j}+7\mathbf{k})$

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2) Suppose $A=\begin{pmatrix}1 & -3 & 1\\ 2 & 1 & 2\end{pmatrix}$ and $B=\begin{pmatrix}2 & 3\\ x & 0\\ 0 & 2\end{pmatrix}$.

a. If BA is defined, calculate it. Otherwise, simply say that it is undefined.

$$BA=\begin{pmatrix}8 & -3 & 8\\ x & -3x & x\\ 4 & 2 & 4\end{pmatrix}$$

b. If AB is defined, calculate it. Otherwise, simply say that it is undefined.

$$AB=\begin{pmatrix}2-3x & 5\\ 4+x & 10\end{pmatrix}$$

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3) Suppose $A=\begin{pmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ 0 & 0 & x\end{pmatrix}$ and $B=\begin{pmatrix}y & 0 & 0\\ 2 & 1 & 0\\ 1 & 1 & 3\end{pmatrix}$.

a. Calculate $\det(A)$.

$$\det(A) = +1\det\begin{pmatrix}2 & 1\\ 0 & x\end{pmatrix} - 2\det\begin{pmatrix}0 & 1\\ 0 & x\end{pmatrix} + 2\det\begin{pmatrix}0 & 2\\ 0 & 0\end{pmatrix} = 2x.$$

b. Calculate $\det(B)$.

$$\det(B) = +y\det\begin{pmatrix}1 & 0\\ 1 & 3\end{pmatrix} - 0\det\begin{pmatrix}2 & 0\\ 1 & 3\end{pmatrix} + 0\det\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix} = 3y.$$

c. Calculate $\det(AB)$.

$$\det(AB) = \det(A)\det(B) = 6xy.$$

d. Calculate $\det(BA)$.

$$\det(BA) = \det(B)\det(A) = 6xy.$$
