Worksheet Number 10

In the Jiuzhang Suanshu written about 100 AD, there appears a method for solving systems of several linear equations. This method was known in Europe only in the 1700ís AD, and is the basis of the best algorithms implemented on modern computers today. In linear algebra courses this is called the Gauss-Jordan elimination algorithm.

Here is the example from the book:

You want to solve the system

$$3x + 2y + z = 39$$
$$2x + 3y + z = 34$$
$$x + 2y + 3z = 26$$

First, agree to write only the coefficients, the numbers:

3 2 1 39
2 3 1 34
1 2 3 26

then multiply the top first number, 3, times the whole second row;

3 2 1 39
6 9 3 102
1 2 3 26

Now they subtracted twice the first row from the second, to get:

3 2 1 39
0 5 1 24
1 2 3 26

The idea is to get a lot of useful zeros

Now multiply the 3rd row by the same 3 and subtract the first row from it:

3 2 1 39		3 2 1 39
0 5 1 24	which becomes	0 5 1 24
3 6 9 78		0 4 8 39

Now do the same thing for the last two rows: multiply the 3rd row by the leading 5 in the second row and then subtract 4 times the second row from the 3rd row:

3 2 1 39		3 2 1 39
0 5 1 24	which becomes	0 5 1 24
0 20 40 195		0 0 36 99

Now you read off: $36z = 99$, so $z = 2\ 3/4$, and substitute 2 3/4 in the equation

above to get y: $5y + 2\ 3/4 = 24$, or $5y = 21\ 1/4 = 20\ 5/4$, or $y = 4\ 1/4$.

Use these two values for z and y to get x from the first equation
$3x + 2(\ 4\ 1/4\ ) + 2\ 3/4 = 39$, so $3x + 11\ 1/4 = 39$, so $3x = 27\ 3/4$, so $x = 9\ 1/4$.

Now you find the solution to this system of equations using this method.

$x + 2y + 3z = 1$ If you get a really simple equation, switch it to the bottom line

$2x + 4y + 7z = 2$

$3x + 7y + 11z = 8$