

Worksheet Number Eighteen

Let $S = \{ a, b, c \}$, a set of three elements, the letters “a,” “b,” and “c”.

1. List the $2^3 = 8$ subsets of the set S .

Let $T = \{ a, b, c, d \}$, a set of four elements.

2. List the $2^4 = 16$ subsets of the set T .

3. Find a 1-1 correspondence between the set of natural numbers, \mathbb{N} , and the set of all points in the plane, both of whose coordinates are integers. Like $(2, 5)$, $(-3, 18)$ and $(-35, -57)$.

Here is a different 1-1 correspondence between the natural numbers, \mathbb{N} , and the positive rational numbers \mathbb{Q}^+ . $1 \mapsto 1$, $\frac{1}{2} \mapsto 2$, $\frac{2}{3} \mapsto 12$,

$$\frac{p}{q} \mapsto p^2 \cdot q$$

where p and q are primes, $\frac{p^k \cdot r^l}{q^m \cdot s^n} = p^{2k} \cdot r^{2l} \cdot q^{2m} \cdot s^{2n}$, where p, q, r and s are primes, and so on

4. What rational numbers get matched up with the numbers

1	2	3
4	5	6
7	8	9
	and 10	?

5. What rational numbers get matched up with the numbers

101	102	103
104	105	106
107	108	109
	and 110	?

Let $[0, 1]$ be the set of all real numbers between 0 and 1, inclusive.

That is, $[0, 1] = \{x \text{ any real number so that } 0 \leq x \leq 1\}$.

Let $F =$ all functions from $[0, 1]$ to $[0, 1]$.

Examples of functions from $[0, 1]$ to $[0, 1]$:

$f(x) = .5$ for all $x, 0 \leq x \leq 1$; $f(x) = 1$ for all $x, 0 \leq x \leq 1$

$f(x) = x$ for all $x, 0 \leq x \leq 1$; $f(x) = x^2$ for all $x, 0 \leq x \leq 1$

$f(x) = x^3$ for all $x, 0 \leq x \leq 1$; $f(x) = \begin{cases} .25 & \text{for all } x, 0 \leq x \leq .5 \\ .75 & \text{for all } x, .5 < x \leq 1 \end{cases}$

There are a lot of elements in F . Let's see that the cardinality of F is bigger than c , the cardinality of the set of all real numbers.

6. First give a function which matches nicely with a value of x , so that you can get as many functions as you have real numbers between 0 and 1.

7. Now, suppose we have a 1-1 correspondence of x 's to functions, $x \mapsto f_x$.
How can you make a function not on our list, here?

$f(x) =$