MathU201 Worksheet #2

January 9, 2005 Professor Gilmore

Mesopotamian Calculations

Here is the calculation of $\sqrt{2}$ by the algorithm on page 32 of our text. As you might infer from his wording, we do not know that the Mesopotamians knew this algorithm.

Take the number whose square root you want to find, in our case 2, and guess a square root. We'll guess the number 1 for a0, our starting number.

To get the next number, do this: (a0 + 2/a0)/2 = a1.

$$(1 + 2/1)/2 = (1 + 2)/2 = 3/2 = 1.5$$

Then repeat the same procedure on a1 to get a2: (a1 + 2/a1)/2 = a2.

$$(3/2 + 2/(3/2))/2 = (3/2 + 4/3)/2 = (17/6)/2 = 17/12 = 1.41\overline{6}$$

Then repeat the same procedure on a2 to get a3, and so on, as long as you need to:

$$(17/12 + 2/(17/12))/2 = (17/12 + 24/17)/2 = \left(\frac{17^2+24\cdot 12}{2\cdot 17\cdot 12}\right) = 577/408 = 1.4142156862745$$

and the actual approximation to $\sqrt{2}$ given by a TI-83 calculator is 1.41421356237. So three steps got us very close. One more step and we bypass the accuracy of a twelve-place calculator!

1. $\sqrt{8}$ =

2. $\sqrt{15}$ =

3. $\sqrt{21} =$

4. The Mesopotamians also knew the quadratic formula as long as everything remained as positive numbers in the calculation. They also used completing the square as a method of solving quadratic equations. Use this method to solve the following

$$x^2 + 10 \cdot x = 39$$