

The Thirteen Books of Euclid's Elements

Sketch of contents book by book

Book I Triangles: Proposition 32, the sum of the angles in any triangle is 180 degrees.

Proposition 47, the final proposition in this book, is the
Theorem of Pythagoras.

Proposition 48, the converse of the Theorem of Pythagoras

Book II Areas: Proposition 11 constructs the division of a line segment, \overline{AC} into two segments, \overline{AB} and \overline{BC} , so that $(\overline{AB})^2 = (\overline{BC}) \cdot (\overline{AC})$.
This is called **division into mean and extreme ratio.**

Proposition 14 shows how to construct a square with area equal to any figure formed by a number of straight lines which close up.

Book III Circles, chords, angles in circles:

Proposition 31 proves that an angle inscribed in a semicircle is a right angle.

Book IV Circles and Polygons: Inscribe regular triangles, squares, pentagons (Proposition 11), hexagons and 15-gons (Proposition 16) in a given circle, and also circumscribe circles about given polygons.

Book V Proportions and magnitudes.

Book VI Similar figures.

Book VII Number Theory: **Prime numbers**, the **Euclidean algorithm** to find the GCD

Proposition 2, How to find the greatest Common Divisor of two integers.

Book VIII Large sets of numbers, squares and cubes of integers.

Book IX More on numbers, even and odd numbers.

Proposition 14. **The Fundamental theorem of arithmetic.**
Any integer is represented uniquely as a product of powers of primes

Proposition 20, **There is an infinity of prime numbers.**

Proposition 36, The formula for all even **perfect numbers**,
 $2^{n-1}(2^n - 1)$ if $2^n - 1$ is a prime number.

Book X Irrational Numbers: Proposition 4, To find the greatest common divisor of three commensurable magnitudes.

Lemma 1 after Proposition 28, A method which can give all **Pythagorean triples** of numbers.

Proposition 115. From a given medial line (a line with irrational length) one can construct infinitely many different irrational lines.

Book XI Solid Geometry: Lines, planes, parallelepipeds and solid angles formed by planes.

Book XII Solid Geometry: Volumes of cones, pyramids, cylinders and spheres.

Proposition 2, The ratio of the areas of circles are to each other as the squares of their diameters.

Book XIII Solid Geometry: The **five regular solids** constructed with ruler and compass, inscribed in a sphere, their dimensions carefully compared.

Propositions 13, 14, 14, 16 and 17.