

Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	total
points															

MTHU242 Final Exam

April 22, 2004

Instructor's name _____ Your name _____

For full credit, please show all work. Answers from your calculator, without supporting work, are worth zero points. The total number of points on the exam is 100. Good luck!

1) (5 points each) Calculate the integrals:

a) $\int_1^8 x^{3/2} \ln x \, dx$

b) $\int \frac{(x-29)}{(x+7)(x-11)} \, dx$

2

2)

a) (6 points) Solve the initial value problem $y' = (4t^3 + 1)y^2$, with $y(1) = 0.5$.

b) (2 points) Using your $y(t)$ from part a), determine the value of $y(0)$.

3) (7 points) Find the area of the region bounded by the line $y = 2 - x$ and the curve $y^2 = 4 - x$.

4) (7 points) Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the x -axis.

5) Determine, using an appropriate limit, whether each of the following integrals diverges or, if it converges, calculate the value of the integral.

a) (3 points) $\int_3^{\infty} \frac{9 dx}{1+x}$

b) (4 points) $\int_0^1 \frac{dx}{\sqrt{1-x}}$

6) (7 points) A curve is traced out by the graph of the function $f(x) = 3 + \frac{1}{4}x^{3/2}$. Find the arc length along this curve from $x = 0$ to $x = 4$.

4

7)

a) (2 points) Sketch the polar region bounded by the spiral $r = \theta$ and the rays $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$.

b) (5 points) Find the area of the region from part a).

8) (7 points) Write out a Simpson's Rule estimate with $n = 6$ for $\int_0^{1.2} \sin(\pi x^2) dx$. Write out the individual terms explicitly, like $\sin(\pi(0.75)^2)(0.01)$, but **DO NOT EVALUATE**.

9) (7 points) Find the values of x for which the geometric series $\sum_{k=1}^{\infty} (2x - 1)^k$ converges.

10) We know that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{k=0}^{\infty} x^k, \quad \text{provided that } |x| < 1.$$

a) (5 points) Fill in the blanks.

By using the power series above, we find the following power series, centered at 0, for $\frac{1}{1+5x}$:

$$1 + \text{_____} x + \text{_____} x^2 + \text{_____} x^3 + \dots = \sum_{k=0}^{\infty} \text{_____} x^k.$$

b) (2 points) Fill in the blank.

The series in part a) actually converges and equals $\frac{1}{1+5x}$ provided that $|x|$ _____.

11) (7 points) Find the radius of convergence and the center of the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n(x+2)^n}{n}$.

Radius of convergence= _____.

Center of the interval of convergence= _____.

12) (7 points) Find a unit vector that is perpendicular to both $\vec{a} = \langle 3, 2, 0 \rangle$ and $\vec{b} = \langle 3, 1, 5 \rangle$.

13) (3 points each) Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 5 \end{pmatrix}$. Find the following (if undefined, simply state so):

Show a couple of the sums that you use in your calculations.

a) AB

b) BA

14) (2 points each) Fill in the blanks. Show your work when you calculate the determinants, and clearly indicate any rule that you use.

Suppose that $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$.

$\det(A) = \underline{\hspace{2cm}}$

$\det(B) = \underline{\hspace{2cm}}$

$\det(AB) = \underline{\hspace{2cm}}$.