Practice Final

MTHU 242 **Spring 2007**

TO OBTAIN CREDIT YOU MUST WRITE OUT ALL INTERMEDIATE STEPS CONNECTING THE STATEMENT OF THE PROBLEM TO YOUR SOLUTION. A solution which is not satisfactorily justified (for example the computation of an integral or a limit with missing intermediate steps) will be regarded as void and will not be given **any** credit.

Name:

(1) Evaluate the following integrals

(a) $\displaystyle\int_{\frac{2}{3\pi}}^{\frac{2}{\pi}} \frac{\cos(\frac{1}{x})}{x^2}\, dx$

(b) $\displaystyle\int x^2 e^x dx$

(c) $\displaystyle\int \cos^3(x)\sin^2(x)\, dx$

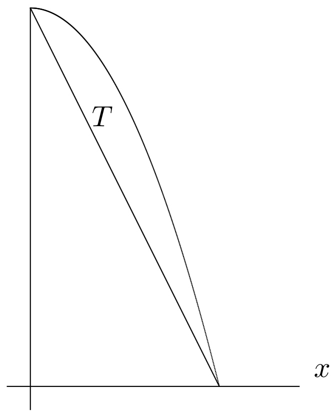
(2) Use Simpson's rule based on 5 intervals to set up a numerical approximation to $\int_0^1 \sin(x^2)\, dx$. **Do not evaluate this approximation numerically**.

(3) Determine whether the following integrals are convergent. When they are, compute their value. Show any work on limits explicitly.

(a) $\displaystyle\int_1^\infty \frac{dx}{\sqrt[7]{x^5}}$

(b) $\displaystyle\int_0^\infty \frac{3dx}{1+x^2}$

(4) Let T be the region bounded by the curve $y = 4 - x^2$ and the line $y = 4 - 2x$. See the graph below.



(a) Find an integral for the area of T. **DO NOT EVALUATE THE INTEGRAL.**

(b) Set up an integral using the washer method to determine the volume of the solid generated by rotating the region T about the line $y = -1$. **DO NOT EVALUATE THE INTEGRAL.**

(c) Set up an integral using the washer method to determine the volume of the solid generated by rotating the region T about the line $x = 5$. **DO NOT EVALUATE THE INTEGRAL.**

(5) A tank has the shape of an inverted circular cone with height 10m. and base radius 4m. It is filled with water up to a height of 8m. Find the work required to empty the tank by pumping the water from its top. (The density of water is 1000 Kg/m^3). **DO NOT EVALUATE THE INTEGRAL.**

(6) Determine the limit as n goes to infinity of the following sequences:

(a) $a_n = (-1)^n \dfrac{n^3}{2n^3 + 2n^2 + 3}$

(b) $b_n = ne^{-n}$

(7) Determine whether the following series are convergent or divergent. When they are convergent, find their sum. (State which convergence or divergence criterion you are using, if any, and carefully show why it is applicable.)

(a) $\displaystyle\sum_{n\geq 2} \frac{2^n}{5^n}$

(b) $\displaystyle\sum_{n\geq 2} \frac{1}{n \ln n}$

(8) Consider the series $\sum_{n\geq 1}(-1)^{n-1}\frac{1}{\sqrt[3]{n}}$.

(a) Prove that it is convergent.

(b) Determine whether the series is absolutely convergent?

(c) Find a number N such that $\sum_{n=1}^{N}(-1)^{n-1}\frac{1}{\sqrt[3]{n}}$ approximates $\sum_{n\geq 1}(-1)^{n-1}\frac{1}{\sqrt[3]{n}}$ with a precision of 0.1.

(9) Consider the power series $\sum_{n\geq 1} 4^n (x+5)^n$.

(a) Determine its radius of convergence.

(b) Determine its interval of convergence.

(10) Find the Maclaurin series for the function $\cos(x^3)$.

(11) Determine the Taylor series of the function $f(x) = \sin(x)$ at the point $a = \pi/4$.

(12) Consider the vectors $\vec{a} = \langle -1, -1, -4 \rangle$ and $\vec{b} = \langle 2, 3, 6 \rangle$.

(a) Find the vectors $\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

(b) Compute the lengths $|\vec{b}|$ and $|\vec{a} + \vec{b}|$.

(13) Find a unit vector in three–space with the same direction as $3\vec{i} + 2\sqrt{2}\vec{j} + 2\sqrt{2}\vec{k}$.