Mth U242 Spring 2008 **Final Review**

1. Find the antiderivatives:

a) $\int xe^{3x}\,dx$ **b)** $\int \ln(3x)\,dt$ **c)** $\int_{\ln(\pi/2)}^{\ln(3\pi/2)} e^x \sin(e^x)\,dx$ **d)** $\int x^2 \ln(x)\,dx$

e) $\int \frac{\arctan(x)}{1+x^2}\,dx$ **f)** $\int \sin^6(x)\cos^3(x)\,dx$ **g)** $\int \frac{x+7}{x^2-7x+12}\,dx$ **h)** $\int \frac{x^3}{\sqrt{1-x^2}}\,dt$

2. Estimate the following using the Trapezoid and Simpson's methods: $\int_{-1}^{1} 2^{(-x^2)}\,dx$ using n=4

3. Decide if the the following improper integrals converge or diverge and give the value if they converge:

a) $\int_0^\infty x^2 e^{-(x^3)}\,dx$ **b)** $\int_1^2 \frac{x}{\sqrt{x^2-1}}\,dx$ **c)** $\int_0^1 \frac{1}{\sqrt[3]{x^5}}\,dt$ **d)** $\int_0^\infty \cos(x)\,dt$

4. Find the area between the following curves:

a) $y = 8 - x^2$ and $y = x^2$ from x=0 to x=3 **b)** $y = x^2 - 4$ and $y = 2 - x$

c) $y = x^2$, $y = 2x$,

5. Find the volume of the solid you get if (set up only)

a) the region between $y = x$ and $y = x^2$ is revolved about the x=−1 (or x=5).

b) the region between $y = x$, $y = 2x$, x=1, and x=2 is revolved about the x-axis (or y=−2).

c) the region between $y = x^2$ and $y = 2x$ is revolved about the y-axis. (set up the integral using both methods).

6. Find the arc length of the following:

a) the curve defined by $x(t) = 2\sin(t)$, $y(t) = 3t$, $z(t) = 2\cos(t)$ from (0,0,2) to (2,$\frac{3\pi}{2}$, 0)

b) the curve $y = \frac{2}{3}(x-1)^{3/2}$ from x=1 to x=4

7. a) A cylindrical tank is 10m high and has a radius of 7m. It is half filled with water. Set up an integral to find the work required to empty out the tank from its top (the density of water is 1000kg/m^3).

b) A spring of natural length of 2ft has the spring constant 40 lbs/ft. Calculate the amount of work done when the spring is stretched from a length of 2.5 to 3 ft.

c) A conical tank is 20 feet high and has a radius of 5 feet at the base with its vertex 3 feet below ground level. If the tank is full of water weighing 65 lb/ft^3, set up an integral to find the work done in pumping all the water to ground level.

8. Determine whether the following sequences converge or diverge and if it converges, find the limit:

a) $a_n = (-1)^n \frac{n^4}{5n^3+n}$ **b)** $a_n = \frac{\ln(n)}{n}$ **c)** $a_n = \frac{3^n}{n!}$

9. Determine whether the following series converge or diverge (you should also be able to say if a series is absolutely convergent and find the sum for geometric series):

a) $\sum_{n=0}^{\infty} \frac{3^{n+2}}{4^n}$ **b)** $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n+1}$ **c)** $\sum_{n=1}^{\infty} ne^{-n}$ **d)** $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}$

e) $\sum_{n=0}^{\infty} \frac{5}{3n^2+2n+1}$ **f)** $\sum_{n=2}^{\infty} \frac{1}{8n+7}$ **g)** $\sum_{n=2}^{\infty} \frac{(-1)^n}{8n+7}$ **h)** $\sum_{n=2}^{\infty} \frac{(-1)^n n}{8n+7}$

i) $\sum_{n=1}^{\infty} \frac{2}{n^4}$ **j)** $\sum_{n=1}^{\infty} \frac{1}{n(\ln(n))^3}$

10. Find the radius of convergence and interval of convergence for:

a) $\sum_{n=1}^{\infty} 2^n(x+1)^n$ **b)** $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n+1)^2}$ **c)** $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{2^{n+1}}$

d) $\sum_{n=1}^{\infty} x^n(n+1)!$ **e)** $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n6^n}$

11. Write the infinite series for the following functions and give the radius of convergence:

a) $f(x) = \frac{1}{1-x}$ **b)** $f(x) = \frac{x}{1+2x^2}$

12. Estimate $\sum_{n=2}^{\infty} (-1)^n \frac{2}{n^2}$ to within .0003.

13. Find the first four nonzero terms of the Taylor series for the following functions:

a) $y = e^x$ centered about a=2 **b)** $y = e^{-(x^2)}$ centered about a=0

c) $y = \int e^{-(x^2)}\,dx$ centered about a=0 **d)** $y = \sin(x)$ centered about $a = \pi$

e) $y = x\cos(\sqrt{x})$ centered about $a = 0$

f) $y = x\cos(\sqrt{x})$ centered about $a = 0$ and estimate $\int_0^1 x\cos(\sqrt{x})\,dx$ and give the error.

14. Decide if the following is the equation of a sphere. If so, give the center and radius:
$x^2 + y^2 + z^2 - 10x + 2y + 5z = -10$

15. If the acceleration of a particle is given by $\bar{a} = -\cos(t)\bar{i} - \sin(t)\bar{j} + \bar{k}$ with the initial postions $\bar{r}(0) = \bar{i}$ and initial velocity $\bar{v}(0) = \bar{j}$, determine the velocity and position functions.

16. Given the vectors $\bar{u} =< 2, 3, -3 >$ and $\bar{v} =< 4, -2, 5 >$, find:

a) $2\bar{u} + 3\bar{v}$ **b)** $|\bar{u}|$ and $|2\bar{u} + 3\bar{v}|$

17. A curve is given by $\bar{r}(t) =< e^t + e^{-t}, 5 - 2t >$ for $0 \le t \le 3$

a) what is $\bar{r}(0)$? **b)** find the unit tangent vector when t=1 (and give the tangent line).

c) set up the integral to find the length of the curve.

18. A force $\bar{F} =< 2, 3, 6 >$ (in Newtons) moves an object on a line from P=(2,2,2) to Q=(3,6,10) (in meters).

a) Find the displacement vector $\bar{PQ}$ **b)** Find the angle between $\bar{F}$ and $\bar{PQ}$.

c) Find the work done by $\bar{F}$ acting through $\bar{PQ}$ **d)** Find $\text{proj}_{\bar{PQ}}\bar{F}$

19. Give the unit tangent vector $\bar{T}$ and the unit normal vector $\bar{N}$for $\bar{r} =< t, t, t^2 >$ at t=3.