

MTH U242, Practice Final '08¹

1. Evaluate the integrals:

(a) $\int \frac{\sin 2x}{1+\cos^2 x} dx$

(b) $\int \sin x \ln(\cos x) dx$

(c) $\int \frac{dx}{\sqrt{1-x^2}}$

(d) $\int \frac{x^3}{\sqrt{1-x^4}} dx$

(e) $\int xe^x dx$

(f) $\int \frac{2x+1}{(x+1)(x+2)} dx$.

2. Determine whether the following improper integrals are convergent:

(a) $\int_0^\infty e^{-x} dx$

(b) $\int_0^1 \ln x dx$

(c) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$.

3. Find the area of the region enclosed by

(a) the line $y = x - 1$ and the parabola $y^2 = 2x + 6$;

(b) $x = 2y^2$ and $x + y = 1$.

4(a) Find the volume of the solid you get if the region enclosed by the parabolas $y = x^2$, $y^2 = 8x$, is rotated about the x -axis.

4(b) The region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is rotated about the line $x = 1$. Find the volume of the resulting solid.

¹preliminary version

- 5(a) A cable that weighs 2lb/ft is used to lift 800 lb coal up a mineshaft 500 ft deep. Find the work done.
- 5(b) The vertical end of a tank is an isosceles triangle with height 3 m and base 3 m. Assume the tank is 8 m long and that it is full of water and that the water is to be pumped to a height 2 m above the top of the tank. Find the work done in emptying the tank.

6. Find the limits of the sequences:

- (a) $\lim_{n \rightarrow \infty} \frac{n^4+7}{7n^4}$
- (b) $\lim_{n \rightarrow \infty} \frac{(\ln(n))^2}{n^3}$
- (c) $\lim_{n \rightarrow \infty} e^{\frac{\sqrt{n}+1}{n}}$.

7. If the following series converges, give a number to which it converges:

- (a) $\sum_{n=0}^{\infty} \frac{253}{(-3)^n}$
- (b) $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n}$
- (c) $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$.

8. Determine whether the following series are convergent or divergent:

- (a) $\sum_{n=0}^{\infty} \frac{n}{n+1}$
- (b) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$
- (c) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{\sqrt{n^3}}$
- (d) $\sum_{n=1}^{\infty} \frac{nn!}{(n+2)!}$
- (e) $\sum_{n=1}^{\infty} (-1)^n \frac{2}{n+5}$.

9. Determine the interval of convergence and the radius of convergence of the power series:

- (a) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)3^n}$
- (b) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$.

Problem 10. Let $f(x) = \frac{x^2}{4+x^2}$, $-2 < x < 2$.

- (a) Find a power series representation for f
- (b) Find a power series representation for $\int \frac{x^2}{4+x^2} dx$
- (b) Find a power series representation for f' .

11. Find the Taylor series of $f(x)$ at a

- (a) $f(x) = \cos x$, $a = \pi$
- (b) $f(x) = e^{3x}$, $a = 0$
- (b) $f(x) = \sin x$, $a = \pi/2$.

12. Consider the vectors $\vec{a} = \langle 1, 2, 3 \rangle$ and $\vec{b} = \langle -1, -2, 3 \rangle$.

- (a) Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.
- (b) Find the dot product $\vec{a} \cdot \vec{b}$.
- (c) Find the angle between \vec{a} and \vec{b} .

13. Find a vector equation and parametric equations for the line connecting the points $P(1, -1, 2)$ and $Q(5, 2, 8)$.

14. Let $P(4, 3, 5)$, $Q(1, 1, 1)$ and $R(-1, 10, -2)$ be the vertices of the triangle $\triangle PQR$.

- (a) Find the angle at P ;
- (b) Is it true that the angle at Q is right? If "Yes", what is the length of the hypotenuse?

15. Consider the vector function $\vec{r}(t) = \langle 1, t^2, t^3 \rangle$, $0 \leq t \leq 1$.

(a) Find the derivative $\vec{r}'(t)$

(b) Find the length of the curve.

16. Find the unit tangent and normal vector to the curve $\vec{r}(t) = \langle \sin 3t, \cos 3t, t \rangle$ at $t = \pi/9$.