

Name: _____.

Show All Your Work
Answers from a calculator receive no credit

1. (8 points) Solve the initial value problem:
 $36y'' - 12y' + y = 0$ with $y(0) = 3$ and $y'(0) = 5/2$.

2. Find the general solution to each of the following equations:

a. (8 points) $xy' = y + xe^{\frac{y}{x}}$

b. (9 points) $y'' + 7y' + 12y = 4e^{2x} + 5e^{4x}$

3. (4 points) **Set up, but DO NOT SOLVE:**

A chemical manufacturing plant has a 1,000-gallon holding tank, which it uses to control the release of pollutants into a sewerage system. Initially the tank has 360 gallons of fluid in it, containing 2 pounds of pollutant per gallon. Fluid containing 3 pounds of pollutant per gallon enters the tank at the rate of 80 gallons per hour, and is uniformly mixed with the fluid already in the tank. Simultaneously, fluid is released from the tank at the rate of 40 gallons per hour. Set up the differential equation to find the function, $P(t)$, giving the amount of pollutant in the tank at time t .

DO NOT SOLVE THIS DIFFERENTIAL EQUATION.

4. (8 points) Are the following vectors linearly independent or are they linearly dependent? Give your reasons and explain your work:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 7 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 3 \\ 3 \\ 4 \end{bmatrix} \quad v_3 = \begin{bmatrix} 2 \\ 0 \\ 5 \\ 1 \end{bmatrix}.$$

5. (8 points) Find all solutions to the following system of equations, and express this answer in vector form: a constant vector plus the non-constant vectors multiplied by the arbitrary scalars.

$$2x_1 + x_3 - x_6 = 1$$

$$x_2 - x_3 + x_5 = 2$$

$$x_5 - x_6 = 3$$

6. (9 points) Find the inverse Laplace transform of $F(s) = \frac{3s + 5}{s^2 + 2s + 1}$.

7. (9 points) Use the Laplace transform to solve the initial value problem:
 $y'' + 4y = u(t - 2)$ with $y(0) = y'(0) = 0$.

8. (8 points) A spring with constant $k = 68$ lbs/ft is attached to a mass, $m = 2$ slugs, and also to a dashpot with damping constant $c = 12$ lbs/(ft/sec). There is no external force acting. If the spring is set in motion with an initial position of $x(0) = -2$ ft, and an initial velocity of $x'(0) = 16$ ft/sec, find the position function, $x(t)$.

9. (4 points) **DO NOT CALCULATE THE DETERMINANT:**
Given the matrix below, set up the determinant used to find the eigenvalues of A, **BUT DO NOT CALCULATE THIS POLYNOMIAL**

$$A = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 4 \\ 6 & 0 & 3 & 12 \\ 2 & 0 & 0 & 5 \end{bmatrix}$$

10. (8 points) Given that the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ are 2 and 1, find the eigenvectors of this matrix, working by hand.

11. You are given the eigenpairs $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ of the matrix

$$A = \begin{bmatrix} 7 & 6 \\ 2 & 10 \end{bmatrix}.$$

- a. (2 points) Find D , the diagonalization of A , and the non-singular matrix, P , that can be used, together with its inverse, to get from D to A . Write these two matrices down.
- b. (4 points) Find P^{-1} by hand.

- c. (2 points) Express A^7 as a product, using P, D, and P^{-1} .
Do not multiply this product out.

12. (9 points) Given the 2X2 system of differential equations
- $$\begin{aligned} \dot{x}_1 &= x_1 - 5x_2 \\ \dot{x}_2 &= x_1 + 3x_2 \end{aligned}$$
- with the eigenpairs $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2i, \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 2i$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2i, \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 2i$ given to you,
 find the general solution to this system of equations.