Northeastern University | MthU343

DiffEQs & Linear Alg. for Engineering | Aug. 20, 2007

Final Exam | 2 Hours

Name:______________________.

Show All Your Work

1. (20 points) Solve the following differential equations:

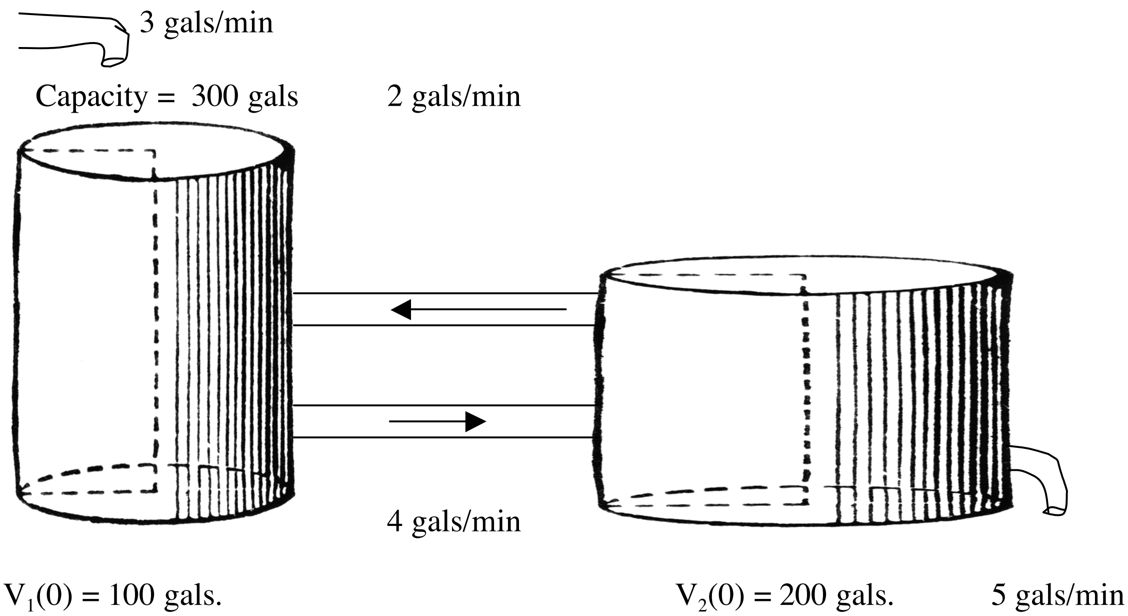
a. $2xyy' = x^2y + xy^2$

b. $x^2y' + 2xy = 5y^4$

c. $y' = -5y + 3x^2e^{-5x}$

d. $\frac{dy}{dx} = \frac{x^2}{(x^3 + 1)(y + 1)}$ (Do not solve for y explicitly.)

2. (5 points) Set up **BUT DO NOT SOLVE** a system of ordinary differential equations for the following problem. There are two tanks of liquid, one containing 100 gallons of brine and the other containing 200 gallons of brine at time t = 0. The first tank has a capacity of 300 gallons. A pipe supplies a constant stream of fresh water pouring into the first tank at a rate of 3 gallons per minute. The mixture in the first tank is piped into the second tank at a rate of 4 gallons per minute. There are two pipes leading out of the second tank. One sends the mixture in the second tank back into the first tank at a rate of 2 gallons per minute, while the other sends the mixture in the second tank out of the system at a rate of 5 gallons per minute.



3. The differential equation $x'' - 10x' + 21x = 0,\ x_0 = 0\ \mathit{and}\ x_0' = 2$ describes the motion of a mass attached to a spring and a dashpot.

a. (8 points) Find the position function x(t).

b. (2 points) Determine whether the motion is over damped, critically damped or under damped. Explain your answer in physical terms.

4. (10 points) The general solution of the differential equation $x'' + 4x' + 4x = 0$ is x(t) = $c_1 e^{-2t} + c_2 t e^{-2t}$. Now continue, using the method of undetermined coefficients, to solve the initial value problem $x'' + 4x' + 4x = 3t + e^{-2t}$ with x(0) = 1 and $x'(0) = 5$. (Warning: c_1 *and* c_2 come at the end.)

5. (10 points) Use Laplace transforms to solve the initial value problem:

$$x'' + 25x = f(t) \quad where \quad f(t) = \begin{cases} 3 & for\ t \geq 2 \\ 0 & for\ 0 \leq t < 2 \end{cases},$$

and x(0) = 5 and $x'(0) = 25$.

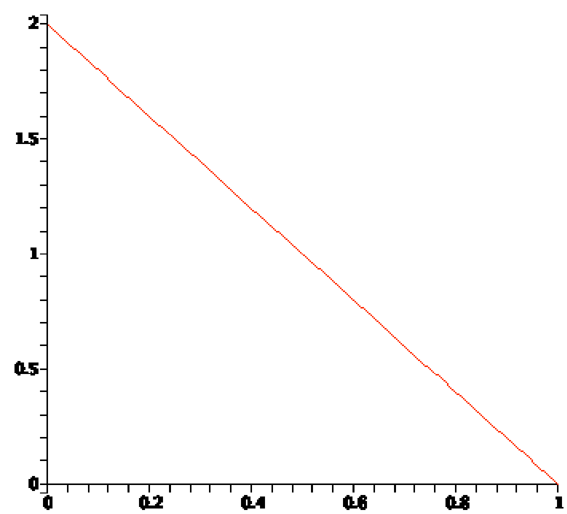
6. (3 points) Determine whether the vectors (1, 0, 1, 3), (1, 3, 0, 1), (1, 0, 7, 6) and (1, 1, 4, 5) are linearly independent. Give reasons for your answer. Show the matrix you use.

7. a. (8 points) Find a basis for the solution space of the homogeneous linear system AX = 0, where $A = \begin{pmatrix} 1 & 2 & 7 & -9 & 31 \\ 2 & 4 & 7 & -11 & 34 \\ 3 & 6 & 5 & -11 & 29 \end{pmatrix}$. Show a beginning and an ending matrix and your reasoning from that matrix.

b. (2 points) What is the dimension of the solution space? Give your reasons.

8. (6 points) Find the eigenvalues of the matrix $A = \begin{pmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$, working by hand.

9. a. (4 points) Find the Laplace transform of the function given by the graph below:



b. Find the Laplace transform of the function given by

$$f(t) = \begin{cases} e^{-5t} & for\ 0 \le t < 5 \\ f(t+5) = f(t) & for\ 5 \le t \end{cases}$$

10. (10 points) Find the eigenpairs of the matrix $A = \begin{pmatrix} 3 & 4 & -2 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, working by hand.

11. Given that the eigenpairs of the matrix $A = \begin{pmatrix} 3 & 0 & 1 \\ 9 & -1 & 2 \\ -9 & 4 & -1 \end{pmatrix}$ are $\left(3, \begin{pmatrix} 4 \\ 9 \\ 0 \end{pmatrix}\right)$,

$\left(-1 + i, \begin{pmatrix} 4 + i \\ 2i - 9 \\ 17 \end{pmatrix}\right)$, *and* $\left(-1 - i, \begin{pmatrix} 4 - i \\ -9 - 2i \\ 17 \end{pmatrix}\right)$,

a. (5 points) Use this information to find the general solution to the system of differential equations: $\vec{x}'(t) = \begin{pmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{pmatrix} = \begin{pmatrix} 3x_1 + x_3 \\ 9x_1 - x_2 + 2x_3 \\ -9x_1 + 4x_2 - x_3 \end{pmatrix}$

b. (3 points) If you know that $x_1(0) = 0, \;\; x_2(0) = 0 \;\; and \;\; x_3(0) = 17$
Solve the resulting initial value problem.