

MthU343
Northeastern University
DiffEQs & Linear Alg. for Engineering
30 Minutes

Professor Gilmore
Aug. 16, 2007
Quiz #6

Name: _____.

Show All Your Work

1. Calculate the eigenvalues of the following matrix, working by hand: $\begin{pmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{pmatrix}$.

2. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ are 1, 2 and 3. Find the eigenvectors that correspond to each of these eigenvalues.

3. The matrix $B = \begin{pmatrix} 5 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 5 \end{pmatrix}$ has eigenpairs $\left(2, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$, $\left(6, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right)$ and $\left(9, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right)$.

a. Use this information to write down the general solution to the linear system of differential equations:

$$x_1'(t) = 5x_1(t) + x_2(t) + 3x_3(t)$$

$$x_2'(t) = x_1(t) + 7x_2(t) + x_3(t)$$

$$x_3'(t) = 3x_1(t) + x_2(t) + 5x_3(t)$$

b. If, in addition you know that $x_1(0) = 1$, $x_2(0) = 2$ and $x_3(0) = 3$, find the exact solution for the resulting initial value problem. You can use a calculator, but write down what you did, including an appropriate matrix.

4. Write the following system of differential equations in matrix form:

$$x_1'(t) = 2x_1(t) - x_2(t) + x_3(t) + 3$$

$$x_2'(t) = -x_1(t) + 4x_2(t) + e^t$$

$$x_3'(t) = 3x_2(t) - 7x_3(t) - \cos(t)$$