

MthU343

Northeastern University

DiffEQs & Linear Alg. for Engineering

65 Minutes

Professor Gilmore

July 24, 2008

Hour Exam

Name: _____.

Show All Your Work

1. Solve the differential equation : $3xy^2y' + y^3 = e^{-x}$ with $y(0) = 1$.

2. The half-life of radioactive cobalt is 5.27 years. A nuclear accident has left a certain region with 100 times the level of radioactive cobalt that is judged safe for human living. How long will it be until that region is again safe for habitation?

3. A car starts from rest with its engine providing a constant acceleration of 10 ft/sec^2 and air resistance creating $.1 \text{ ft/sec}^2$ of deceleration for each foot per second of the car's velocity.

a. Write a differential equation describing this situation.

b. Solve this equation for the velocity of the car as a function of time, and find the limiting velocity of the car.

c. How long will it take for the car to reach 90% of its limiting velocity?

4. Find the Laplace transform, $\mathcal{L}\{f(t)\}$, of the following functions of t :

a. $f(t) = e + t^4 - e^{3t} \cdot \sin(2t)$

b. $f(t) = \begin{cases} t - 7 & \text{for } 7 \leq t < \infty \\ 0 & \text{otherwise.} \end{cases}$

5. A lead cube with a 3 kilogram mass is hanging from a vertical spring set up in a lab in Boston. This spring is stretched 25 centimeters by a force of 9 Newtons. At time $t = 0$ the cube is pulled one meter downwards and set in motion with an initial velocity upwards of 5 meters per second.

a. Write down the differential equation.

b. Is there an external force on this system? What is it?

6. Find the inverse Laplace transforms, $\mathcal{L}^{-1}\{F(s)\}$, of the following functions of s :

a.
$$F(s) = \frac{s + 3}{(s + 2)^2 + 25}$$

b.
$$F(s) = \frac{e^{-3s}}{(s - 7)^2 + 64}$$

7. a. Use the Laplace transform method to solve the following differential equation:
 $x'' - 2x' + x = \delta_3(t)$ with $x(0) = 0 = x'(0)$.

b. Draw the graph of your solution.

8. Solve the differential equation using the method of undetermined coefficients:
 $3x'' + 12x = 20t$ with $x(0) = 1$ and $x'(0) = -5$