Math 1108 (all sections) WS #4 Spring 2000 Solutions

1. A cup of mint tea at 58°C is brought into a room whose temperature is 18°C; after 5 minutes its temperature is 40°C.

A. Find y(t), its temperature at time t, assuming y satisfies NLC

B. At what time is the temperature of the tea 95°F = 35°C?

C. How fast is it cooling when its temperature is 35°C?

D. The same cup of tea is cooled to 5°C, then again brought into the room. After five minutes, what is its temperature?

A. r=18, find A: 58=18+A, A=-40. Find k: 40=y(5)=18+40e^{-k5},

$$=-5k=\ln\left(\frac{40-18}{40}\right)=-.598,\quad k=0.1196.\quad Ans:\ y(t)=18+40e^{-0.1196t}$$

B. Solve $35=18+40e^{-0.1196t}$, $t=7.154\ hrs.$ **C.** $y'=k(r-y)=0.1196(18-35)=-2.033\ °C/hr.$

2. A medicine is given intravenously at a fixed rate of 5 mg/hour it is eliminated from the body at a rate 2y mg/hour. If the initial amount in the body is 1 mg, when will the level reach 2 mg? ***Use*** y' = c-ky = k(c/k-y) and NLC. y'-5-2y=2(2.5-2y), so r=2.5. Solution to NLC: y=2.5+Ae^{-2t}, Find A: y(0)=1=2.5+A, A=-1.5, so y=2.5-1.5e^{-2t}. Solve 2=2.5-1.5e^{-2t}, get t = 0.549 hrs.

3. A lake can support a stable population of 2600 trout. Assume that the number y(t) of trout at time t months after April 1 1999, satisfies the logistic equation y' = ky(r-y). 3A. If 200 trout are introduced on April 1, 1999, and there are 600 trout on June 1, 1999, find y(t). 3B. When are there 2000 trout?3C. Find the rate of change y' when y=2000 trout. 3D. Give two assumptions of this model.

Find A: $200=\frac{2600}{1+A}$, $1+A=13$, $A=12$.Find c:600=$\frac{2600}{1+12e^{-c2}}$, $1+12e^{-c2}=\frac{2600}{600}$,

$$12e^{-c2}=4.333-1,\ -c2=\ln(3.333/12),\ c=.640,\ Ans:\ y=\frac{2600}{1+12e^{-.640t}},$$

3B. Solve y(t)=2000, t=5.76mos. **3C.** $y'=ky(r-y)=\frac{.640}{2600}(2000)(600)=295\,fish/mo.$

3'. Using the same data as in #3 (except that of stable population), find when there are 2000 trout using an exponential growth model. y=A$_0$e^{kt}=200e^{kt}, Find k: 600=200e^{k2}, k=(ln(600/200))/2=0.5ln(3)=0.549. When are there 2000 fish? 2000=200e$^{.549t}$, t=(ln(10))/.549=4.19 months.

4. Solve the following by separating the variables, and find a solution through (2,5). (This is an IVP).

A. y'x = y^2. B. y'y$^{1/3}$=sin(2x).

4A. $\frac{dy}{dx}x=y^2,\ \ y^{-2}dy=x^{-1}dx,\ \int y^{-2}dy=\int x^{-1}dx,\ \frac{y^{-1}}{-1}=\ln|x|+C, C=\frac{5^{-1}}{-1}-\ln 2,\quad \frac{y^{-1}}{-1}=\ln|x|-.893$

4B. $\frac{dy}{dx}y^{1/3}=\sin(2x),\ \ y^{1/3}dy=\sin(2x)dx,\ \int y^{1/3}dy=\int\sin(2x)dx,\ \frac{y^{4/3}}{4/3}=\frac{-\cos(2x)}{2}+C,$

$$C=\frac{5^{4/3}}{4/3}+0.5\cos(4),\quad Ans.\ \frac{y^{4/3}}{4/3}=\frac{-\cos(2x)}{2}+6.086$$

Solution of y' = k(r-y), y = r+Ae^{-kt} (Newton's law of cooling- NLC)

Solution of y' = ky: y = Ae^{kt} (exponential growth/decay see

Solution of y' = ky(r-y), y = r/(1+Ae^{-rkt}), A ≠ 0, or y = 0, y = r.