Math U575 Prof. A. Iarrobino, Jan 31, 08

Group theory Project topics passed in as of January 31, 08

-Students who responded requested that names not be posted just now, as topics have not been definitely chosen. The number (n) after is the number of students passing in a given topic no number = 1 student. My comments in italics.

The challenge is to find a "reasonable" topic, interesting to you that you can

a; report on with some mathematical content/depth (related to group theory*), that is*

b. delimited, or focused enough., *and*

c. accessible – that you can understand enough to report back on it in a month or so.

Friezes, wallpaper gouprs (4-5)

Is there a way to apply this, or extend to 3D,2D material is somewhat in text

Plane crystallography

Same as or closely related to wallpaper groups Nice to see applicatons. What about 3D crystals?

Group theory in Chemistry, ``Point groups": (2)

Can be very related to above topics. There can be other applications also to DNA decoding and structure.

Tesselations

Tilings of the plane, more general than wallpaper groups. Often related to Escher's art/ A related topic is Grothendiecks ``Dessins d'enfants" (children's drawings).

Groups in PDE

Being invariant under the action of some symmetry group, greatly affects the solutions.

Multithreads (computing)

{*is there group theory in this?)*

Quantum computing?

Same question, but there is a large field of quantum groups. Several experts in Math Dept (as Profs. Chris King, Valerio Toledano Laredo).

Rubik's Cube (2-3)

There are local experts (Prof. Gene. Cooperman in CS). What about Analogous puzzles?

Homology groups:

Very active area, many Math Dept members use them, or work with them: Prof. Alex Suciu.

Basics are relatively accessible.

Braid groups:

Particular groups that occur in many areas: homology, hyperplane arrangements, ...

Prof. Alex Suciu is an expert. A specific topic with lots of connections – a good choice.

Topological groups

(*The group is a topological space: it would help to know some point-set topology first)*

Lie groups (5)

This is a highly developed area, and it would be important to pick an accessible, delimited piece of it. to report on. Several math dept members spcialize in or use it (Prof. Donald King).

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Sl_2(Q), Sl_n (2)
Special linear group of matrices with determinant one, under product. Connections to partitions, to Lie theory.

Permutation groups:
A main topic of course. So one would look for an application (say, action on polynomial rings, or an aspect not studied in the course --- as representations of S_n.

Nilpotent groups,
A subclass of solvable groups. It would be useful to make some application of it, as to Galois theory. Galois group would be another related topic.

Cryptography

a. public key algorithms (3-4)
So how does one relate it to groups? Integers mod a large prime does come in.
b. elliptic curves in cryptography
the elliptic curve has a group structure

Simple groups:
They have no normal subgroups. Building blocks for other groups. A huge theory (3000 pages of journal articles has classified them: but are they correct?. Dan Gorenstein, one of the leaders of the classification was for a time a NU math dept professor. Some good summaries in AMS Notices, ...

Tarski Monster
There are a number of very large groups, some coming up in the classification of simple groups above. The issue would be to describe the mathematical role such a group plays. Why is it of interest to mathematicians?

.Nilpotent orbits:
Refers to the orbit of a nilpotent matrix B under conjugation in Gl(n): the orbits depend only on the partition P of n giving the Jordan blocks of B. Very developed field, but basic ideas are about similarity of matrices.
(Potential related undergrad project):
Some colleagues and I are working on the following question: Which nilpotent nXn matrices A commute with a given nilpotent matrix B in Jordan form, whose Jordan blocks are given by a partition P? Given B, what is the biggest partition Q(P) giving the Jordan blocks for such an A? Surprisingly open question, with connections to Lie theory.
A doable project for some might be to read parts of Panyushev: " Two results on the centralizers of nilpotent elements" and explain it in the simplest cases (Sl_n), Another might be to write a program that does some of the computation of examples, and see what conclusions you can come to from the examples. (Some relation to PDE).