

Math U575: A few questions (math-preparation for U575)
I'm interested in where we are, in terms of preparation. Note that Linear Algebra Math U371 is listed as a prerequisite. Please complete what you can of the following. Although I will go over these, your work on these today is not part of the grade for the course.

A. Define "rational number" carefully from knowledge of the integers. Include a criterion for two rational numbers so defined to be equal.

B. Find the product $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 5 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$.

C. Give a criterion in terms of the entries for the 2x2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to be invertible.

D. Give a criterion for the 2x2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to satisfy $A^3 = 0$ (so A is nilpotent).

E. Show, using mathematical induction, that $(1 + 2 + \dots + n) = \frac{n(n+1)}{2}$.

F. Show that if an $n \times n$ A has a right inverse B, so $AB = I_n$, then also $BA = I_n$. Here I_n is the $n \times n$ identity matrix.

G. Find the kernel of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ acting on in \mathbf{R}^2 as left multiplication of column vectors.
(This is just the nullspace of A).

H. Describe the group of symmetries of a triangle, square. (we'll study this today).