problem	1	2	3	4	5	6	7	8	9	10	11
score											

MTHU141, Fall 2006

Final Exam

Name: ____________________

There are 11 problems. The first 5 problems and the best 5 out of problems number 6 to 11 will be counted. Each problem is worth 10 points, but you must show all of your work for credit. Display at least 3 digits after the decimal for numbers in decimal format. A calculator is necessary. A table of formulas is provided. You have 2 hours. Good luck.

1. Differentiate the following functions.

(a) $y = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

(b) $y = \ln\left(\dfrac{x-1}{x^2}\right)$

(c) $y = \dfrac{1}{x^3 + x^2 + x + 1}$

(d) $y = e^{3x}\cos 2x$

(e) $y = (x^2 + x + 1)^{\frac{2}{3}}$

2. Given the function $f(x) = x^4 - 18x^2 + 25$,

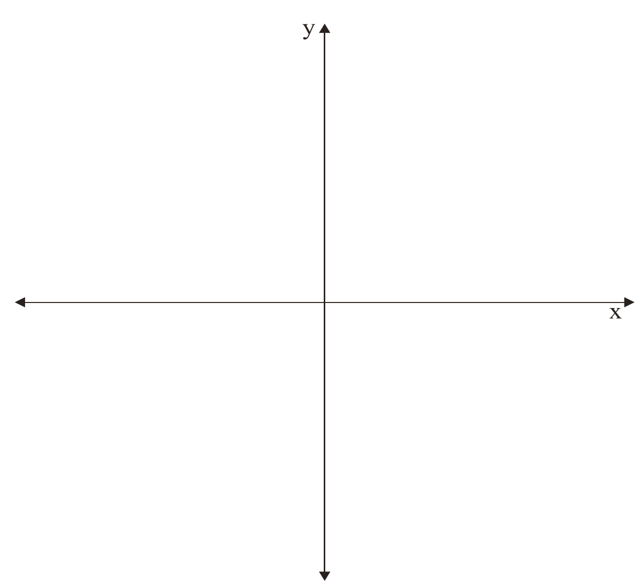
(a) Find its first derivative.

(b) Find its critical points.

(c) Check its critical points for maxima and minima.

(d) Use the second derivative to find its inflections points.

(e) Sketch its graph.



3. Evaluate each of the following integrals.

(a) $\int (x^2 + 5x + 2)\,dx$

(b) $\int \left(\frac{x}{8} + \frac{8}{x}\right)\,dx$

(c) $\int (2\sin x + 3\cos(5x))\,dx$

(d) $\int_0^3 (4\sqrt{x} + 2e^{2x})\,dx$

4. Ice is melting on the surface of a pond at a rate of $-\frac{t^{\frac{2}{3}}}{3}$ cm/hour. At time $t = 0$ the ice is 9 cm thick.

(a) At what rate is the thickness of the ice decreasing at $t = 3$ hours?

(b) How thick is the ice at $t = 3$ hours?

5. Water flowed into a tank at an increasing rate of $r(t)$ from $t = 1$ to $t = 3$ minutes. The rate of flow $r(t)$ was measured at half-minute intervals with the following results.

t in min	1.0	1.5	2.0	2.5	3.0
$r(t)$ in m^3/min	5.00	5.75	7.00	8.75	11.00

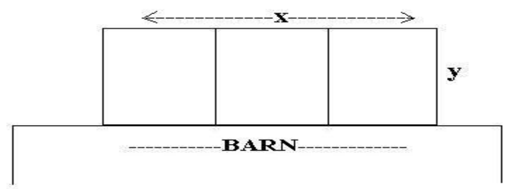
(a) Find the value of the Riemann sum using left endpoints for the total amount of water that flowed into the tank during the time interval $1 \leq t \leq 3$.

(b) Find the value of the Riemann sum using right endpoints for the total amount of water that flowed into the tank during the time interval $1 \leq t \leq 3$.

(c) The exact flow is $r(t) = t^2 - t + 5$ m^3/min. Find the exact amount of water that flowed into the tank during the time interval $1 \leq t \leq 3$.

THE BEST 5 OUT OF PROBLEMS 6 TO 11 WILL BE COUNTED.

6. Three adjacent pens with a total area of 1350 square feet are to be fenced against a barn. (No fence is needed on the barn side.) If exterior fencing costs \$3/foot and interior fencing costs \$1/foot, then find the dimensions x (the total width of the three pens) and y (the depth of the pens perpendicular to the barn) that minimizes the total cost of the fence.



7. The concentration of a drug in a patient's bloodstream t hours after it was injected is given by $C(t) = 16te^{-0.1t}$ mg/ml. When does the concentration reach a maximum and what is that maximum? (Use the first derivative, and show your work.)

8. Eris is a recently discovered dwarf planet located in the Kuiper belt. The current estimate for the acceleration due to gravity near the surface of Eris is approximately -0.60 meters/sec^2. An astronaut standing on a rock 6 meters above the surface of Eris jumps into the air with an initial upward velocity of 2.4 meters/sec. Let t be the number of seconds after the astronaut has started the jump. Use calculus and show your work in the following.

(a) Write a formula for the velocity of the astronaut at time t.

(b) Write a formula for the height of the astronaut above the surface at time t.

(c) When does the astronaut reach the maximum height of the jump?

(d) What is the maximum height of the jump?

(e) When does the astronaut hit the ground?

9. (Two separate and unrelated problems (a) and (b).)

(a) A patient receives an injection of solution containing 90 millicuries (mCi) of fluorine-18 tagged glucose (18F-Fluorodeoxyglucose or FDG) prior to a positron emission tomography scan. You may assume that FDG decays exponentially with a half-life of 110 minutes.

(i) Give a formula for the amount of FDG (in mCi) remaining in the patient t minutes after the injection.

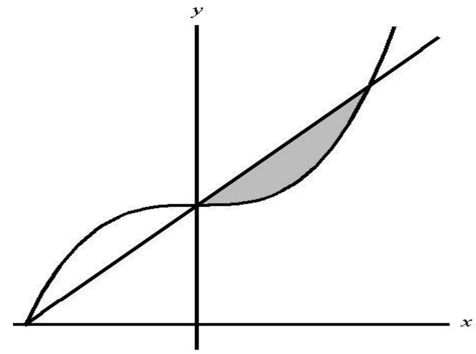
(ii) How long does it take for the amount remaining to be $\frac{1}{8}$ of the original amount?

(b) A fish population left alone increases at a continuous rate of 15% per year. The population is being harvested at a constant rate of 450,000 fish per year.

(i) Write a differential equation for the amount $P(t)$ of fish (in units of 100,000) in t years.

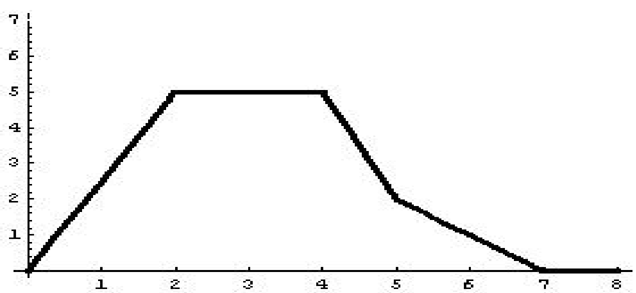
(i) Determine the equilibrium number of fish.

10. Compute the shaded area enclosed by the graphs of the functions $f(x) = x^3 + 1$ and $g(x) = x + 1$ for $x \geq 0$.



11. Oil is leaking from a ruptured tanker at a rate of $r(t)$, in tens of thousands of gallons per hour, where t is in hours after the accident, and the graph of $r(t)$ is as shown.

(a) Write a definite integral giving the total amount (in tens of thousands of gallons) that leaked from the tanker in the first 8 hours after the accident.



(b) Use geometry to compute this total amount.

(c) When did more oil leak from the tanker—in the first 3 hours or between hours 4 and 8? Explain.

FORMULAS

Derivatives
1. $(cf)' = cf'$
2. $(f+g)' = f' + g'$
3. $(fg)' = f'g + fg'$
4. $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$
5. $(g(f))' = g'(f) \cdot f'$
6. $C' = 0$
7. $(x^r)' = rx^{r-1} \quad r \neq 0$
8. $(e^x)' = e^x \quad e = 2.7182818284590\ldots$
9. $(a^x)' = (\ln a)a^x \quad a > 0$
10. $(\ln x)' = \dfrac{1}{x}$
11. $(\sin x)' = \cos x$
12. $(\cos x)' = -\sin x$

Integrals
1. $\int cf\,dx = c\int f\,dx$
2. $\int (f+g)\,dx = \int f\,dx + \int g\,dx$
3. $\int x^r\,dx = \dfrac{x^{r+1}}{r+1} \quad r \neq -1$
4. $\int \dfrac{1}{x}\,dx = \ln|x|$
5. $\int e^x\,dx = e^x$
6. $\int \sin x\,dx = -\cos x$
7. $\int \cos x\,dx = \sin x$

Definition of the Derivative
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Tangent Line to $y = f(x)$ at $x = a$
$$y = f(a) + f'(a)(x-a)$$

Definition of the Integral
$$\lim_{\Delta x\to 0} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)dx$$

Fundamental Theorem of Calculus
If $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Areas
rectangle: $A = \text{(base)(altitude)}$
triangle: $A = \frac{1}{2}\text{(base)(altitude)}$
circle: $A = \pi(\text{radius})^2$
sphere (surface area): $A = 4\pi(\text{radius})^2$

Volumes
cylinder: $V = \text{(area of base)(altitude)}$
pyramid or cone: $V = \frac{1}{3}\text{(area of base)(altitude)}$
sphere: $V = \frac{4}{3}\pi(\text{radius})^3$

Quadratic Formula
$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sines and Cosines
$\sin 0 = 0$	$\cos 0 = 1$
$\sin \frac{\pi}{6} = \frac{1}{2}$	$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos \frac{\pi}{3} = \frac{1}{2}$
$\sin \frac{\pi}{2} = 1$	$\cos \frac{\pi}{2} = 0$

Laws of Exponents
$a^x a^y = a^{x+y}$
$\dfrac{a^x}{a^y} = a^{x-y}$
$(a^x)^y = a^{xy}$
$(ab)^x = a^x b^x$
$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Natural Logarithms
$\ln(xy) = \ln x + \ln y$
$\ln\left(\dfrac{x}{y}\right) = \ln x - \ln y$
$\ln(x^y) = y\ln x$
$e^{\ln x} = x$
$\ln(e^x) = x$
$\ln e = 1$
$\ln 1 = 0$

Motion
$s(t) = \text{distance}$
$s'(t) = v(t) = \text{velocity}$
$s''(t) = v'(t) = a(t) = \text{acceleration}$

Exponential Growth and Decay
$y' = ky \Rightarrow y(t) = Ce^{kt}$ or $y(t) = C(1+r)^t$

Newton's law of Cooling
$T' = k(A - T)$