Math U141 Fall 07 Exam 2 Prof. A. Iarrobino Name____________________

Please show all work, in order to obtain full credit. A graphing calculator may be used. Good luck!

I. ANTIDERIVATIVE, INTEGRAL (90 pts).

1. (25 pts) **Antiderivative:** Find the antiderivatives F of the given functions f(x). Example: f = x^2 has antiderivative $F = \frac{x^3}{3} + C$.

1A. (10 pts) Find the antiderivative F of $4x^3 - x^2$ passing through (3,76) (so F(3)=76).

1B, (15 pts) Find the antiderivative F of f(x) = $\sqrt[3]{x^2} + x^{5/4} + \frac{2}{\sqrt[5]{x^7}} + 3\sin(2x) - e^{3x} + \frac{3}{x} + 2$

2. (20 pts) On a small moon of Jupiter, the gravitational acceleration is -3 m/min^2. Racquel the seal throws a ball with a vertical component of 12 meters/min upward from a hill of height 20 meters above sea level, past a friend Callalah the crow circling at 48 meters above sea level. The ball eventually falls into the sea and a small tiger named Tigoro immediately retrieves it.

A. What is the velocity of the ball at time t minutes after the throw

Ans. v(t)=___________

B. What is the position of the ball at time t minutes after the throw?

Ans. s(t)= _________

C. At what time is the ball highest? How high does it go?

Ans. t= _____, s= ____________

D. Verify that the ball passes Callalah the crow at 6 minutes on the way up. When does the ball pass Calliaah on the way down?

Ans.

E. What is the velocity of the ball just before it falls into the sea?

3. (20 pts) Find the area of the bounded region in the first quadrant enclosed by the curves $y = 4x^2$ and $y = 20x$. (*Hint: first find the intersection points).*

4. (20 pts) The table below gives r(t), the rate of flow of water across a dam, in 10 cubic meters per minute, t minutes after the start of a rainfall.

t min	0	2	4	6	8	10
r(t) $10\,m^3/\min$	7	15	20	30	10	2

4A. Write an integral for the total amount of water that flows over the dam in that period. **Ans.**

4B. Write out the Right R(5), and trapezoid Trap(5) estimates for the total amount of water that flows over the dam in the first ten minutes. In each case, write out the sums, as well as the total. Please include units.

a.Ans. L(6)=

Total: L(6) =

b. Ans. T(6)=

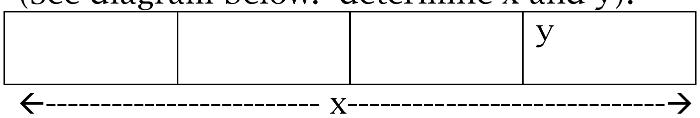
Total: T(6)=

4C. If the flow rate were sampled instead at every 1/20 th second (n=200), determine the difference between the right and left sums for the amount.

Ans R(200)-L(200) =

II. Choose a problem Choose one problem from the following, which will count 20 points (do all parts). You may do a second problem for 5 points.

5. Four rectangular equal pens of total area 520 square feet are built in a row at a cost of \$5/foot for exterior and \$1 foot for interior fencing. Fnd the shape that minimizes the cost of the fencing (determine x,y). Also, show that your answer is
a minimum.
(See diagram below: determine x and y).



6. The velocities of two bicycles, A,B, traveling on a straight path are shown below, over a period of six minutes. The velocity is in feet per second (y-axis), time t is in minutes. At time 0, bike B is at the start of the path, but A begins 20 feet along the path.

The velocity graph for A begins as a straight line between (0,0) and (3,18), then continues as a horizontal line at height 18.

The velocity graph for B is the straight line between (0,20) and (6,-4).

6a. What are the positions of each bike at six minutes?

6b. When is bike B furthest ahead of bike A, and by how much?

6c. What is the average velocity of A in the six minutes?

6d. What is the average acceleration of B in the six minutes?

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Formulas:

Antiderivatives: $\int x^n dx = \frac{x^{n+1}}{n+1}$ +C if n ≠ -1, $\int x^{-1}dx = \ln(x) + \text{C},$ $\int b^x dx = \frac{b^x}{\ln(b)}$ +C.

$\int \sin(ax) \cdot dx = \frac{-1}{a}\cos(ax)$. $\int \cos(ax) \cdot dx = \frac{1}{a}\sin(ax)$, $\int e^{ax} dx = \frac{e^{ax}}{a}$

Quadratic formula: $ax^2 + bx + c = 0$ has solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Area triangle: A=bh/2. Area trapezoid: A= $b(h_1 + h_2)/2.$

Riemann sum formulas for $\int_a^b f(x)dx$, using n equal subdivisions: $\Delta x = (b-a)/n$

L(n) = $(\Delta x)(h_0 + h_1 + h_2 + h_3 + ... + h_{n-1})$ (left); R(n)= $(\Delta x)(h_1 + h_2 + h_3 + ... + h_{n-1} + h_n)$ (right)

T(n) = $(\Delta x)(0.5h_0 + h_1 + h_2 + h_3 + ... + h_{n-1} + 0.5h_n)$ = trapezoid

Note: T(n)=(L(n)+R(n))/2, the average of left and right sums.

NLC: y'=k(A-y) has solution $y = A + Ce^{-kt}$