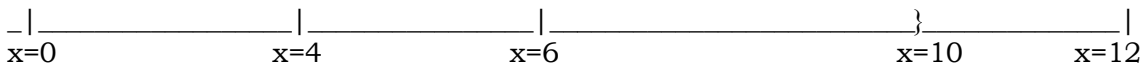


1. The depth of a pollywog in inches below the surface of Houghton Pond is given by  $d(x) = 48x - 4x^2$ ,  $x$  minutes after the beginning of its dive, until it returns to the surface.

- A. When does the pollywog return to the surface?  
 B. What is its average vertical velocity between 60 and 90 seconds?  
 C. Determine the slope function  $d'(x)$  using a quick formula. **Ans.**  $d'(x) =$  \_\_\_\_\_.  
 D. Graph the depth  $d(x)$  as a function of time  $x$ . Also graph  $d'(x)$  on the same axes.  
 $y=150$



E. For what value  $x_1$  of  $x$  is  $d'(x)=0$ ? What is the pollywog doing at time  $x_1$ ? What special point is  $(x_1, d(x_1))$  on the graph of the function  $d(x)$ ?

F. a. Find the equation of the tangent line (TL) to  $y=d(x)$  at  $(7,140)$ . **Ans.** \_\_\_\_\_.

b. For what value of  $x$  does this TL cross the  $x$ -axis? Explain what significance this has for the pollywog (interpret motion along the TL).

H\*. What is a pollywog? (EC for either accuracy, or interesting answer (not just “tadpole”).)

2. Find the derivative functions  $f'(x)$  for the following functions  $f(x)$

2A.  $f(x) = 2x^6 - 3x^4 + 6x^{0.5} - 9$

2B.  $f(x) = \sqrt[4]{x^7} - \frac{3}{\sqrt[5]{x^3}} + \pi x^e$

2C (chain rule) For  $f(x) = (x^4 + 5x^2)^{-3}$  please write  $f(x) = g(u)$ : specify the outside function  $g(x)$  and the inside function  $u(x)$ , then find the derivative  $f'(x)$ .

2D. (chain rule)  $f(x) = e^{-0.7x} - 5e^{-x^2} - 4e^2 + 2\sin(6e^x) + 3e^{\sqrt{x^2-4x}}$

3A. Assume that the function  $f(x)$  satisfies  $f(2)=4, f'(2)=-6$ , and  $f''(2)=8$ . Using approximation along the tangent line, estimate the value of  $f(2.05)$ .

$$f(2.05) = \underline{\hspace{10em}}$$

Also, determine whether your estimate is too high or too low. **Ans.** \_\_\_\_\_  
**because** \_\_\_\_\_.

3B. i. When  $f(x)$  is decreasing, the derivative  $f'(x)$  is \_\_\_\_\_ .

ii. When the derivative  $f'(x)$  is increasing the function  $f(x)$  is \_\_\_\_\_  
 and the second derivative  $f''(x)$  is \_\_\_\_\_.

3C. For the function  $y=f(x)$  given on the board, draw the slope function,  $y=f'(x)$  marking points A' to F' on the slope function corresponding to A-F on the original function.

**Table of derivatives:**

Power rule:  $(x^n)' = nx^{n-1}$ , Extended power rule  $(u^n)' = nu^{n-1} \cdot u'$

Chain rule  $f(u)' = f'(u) \cdot u'$  or  $df/dt = (df/du)(du/dt)$

Exponentials:  $(e^x)' = e^x$ ,  $(e^u)' = e^u \cdot u'$ ;  $(b^x)' = b^x \cdot \ln(b)$ ,  $(b^u)' = b^u \cdot \ln(b) \cdot u'$ ,

Logarithm:  $(\ln x)' = 1/x$   $(\ln u)' = u'/u$

Trig:  $(\sin x)' = \cos x$ ,  $(\sin u)' = (\cos u) \cdot u'$ ,  $(\cos x)' = -\sin x$  (radian measure assumed).

**Other formulas:** Average slope =  $\frac{f(b)-f(a)}{b-a}$ , Instantaneous slope =  $f'(a)$

Exponential growth:  $A = A_0 e^{kx}$