Math U141 Quiz F07 Prof. A. Iarrobino Name: Solutions
Scoring: #1,2 8 pts each, #3 6pts. 20= 100%

1. The depth of a pollywog in inches below the surface of Houghton Pond is given by d(x) = 48x-4x^2, x minutes after the beginning of its dive, until it returns to the surface.

A. When does the pollywog return to the surface? **Sol:** $48x-4x^2=0,\ 4x(12-x)=0,\ x=0,12.$
Ans. *12 min*

B. What is its average vertical velocity between 60 and 90 seconds?

Ans. $\dfrac{f(1.5)-f(5)}{1.5-1}=\dfrac{63-44}{0.5}=38\ in/\min.$

C. Determine the slope function d'(x) using a quick formula. **Ans.** d'(x)=__48-8x__.
D. Graph the depth d(x) as a function of time x. and d'(x) on the same axes. **Ans. in class.**
(***Answer*** *should show d with maximum at x=6 just above where the line y=d'(x) crosses x-axis)*
y=150

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x=0 x=4 x=6 x=10 x=12

E. For what value x_1 of x is d'(x)=0? What is the pollywog doing at time x_1? What special point is (x_1 ,d(x_1)) on the graph of the function d(x)?
Ans. *Set 48-8x=0, x=6 minutes. The pollywog is just changing from descent to ascent. The point P(6,144) is the maximum point of the graph y=d(x).*

F. a.Find the equation of the tangent line (TL) to y=d(x) at (7,140).
Sol. *The slope is d'(7)=48-8(7)= -8 in/min.* ***Ans.*** y-140=-8(x-7) **or** y=-8x+196.
b. For what value of x does this TL cross the x-axis? Explain what significance this has for the pollywog (interpret motion along the TL).
Sol. *Set y=0 in TL eqn: 0=-8x+196, x=24.5 minutes. This is when the pollywog would reach The surface if it continued at constant speed 8 in/min from depth 140 inches at 7 min.*
H*. What is a pollywog? (EC for either accuracy, or interesting answer (not just "tadpole").
Scored +0.5 for answer such as ``larval stage of frog with legs and tail".

2. Find the derivative functions $f'(x)$ for the following functions f(x)

2A. f(x)=$2x^6-3x^4+6x^{0.5}-9$ **Ans.** $12x^5-12x^3+3x^{-0.5}$

2B. f(x)= $\sqrt[4]{x^7}-\dfrac{3}{\sqrt[5]{x^3}}+\pi x^e$ **Sol.** $f(x)=x^{7/4}-3x^{-3/5}+\pi x^e$

Ans $f'(x)=\dfrac{7}{4}x^{3/4}+\dfrac{9}{5}x^{-8/5}+\pi e x^{e-1}$

2C (chain rule) For f(x)= $(x^4+5x^2)^{-3}$ please write f(x) = g(u): specify the outside function g(x) and the inside function u(x), then find the derivative $f'(x)$.

Ans. $g(u)=u^{-3}.\ u(x)=x^4+5x^2.$

$f'(x)=-3\left(x^4+5x^2\right)^{-4}\cdot(4x^3+10x)$

2D. (chain rule) f(x)= $e^{-0.7x}-5e^{-x^2}-4e^2+2\sin(6e^x)+3e^{\sqrt{x^2-4x}}$

Ans. $f'(x)=-0.7e^{-0.7x}+10xe^{-x^2}+12e^x\cos(6e^x)+1.5\left(x^2-4x\right)^{-1/2}(2x-4)e^{\sqrt{x^2-4x}}$

3A. Assume f(x) satisfies f(2)=4, $f'(2)$ =-6, and f "(2)=8. Estimate the value of f(2.05).
Ans. *TL equation:* y-4=(-6)(x-2), y=-6x+16, $y_{TL}(2.05)=4-6(0.05)=4-0.3=3.7.$
The answer is too low, as f(x) is concave up at x=2 (the second derivative f'(2) is positive).
3B. i.When f(x) is decreasing, the derivative $f'(x)$ is *<u>negative</u>* .
ii. When the derivative is $f'(x)$ is increasing the function f(x) is *<u>concave up.</u>*
and the second derivative $f''(x)$ is *<u>negative.</u>* (3C answered in class).

Table of derivatives:

Power rule: $(x^n)' = nx^{n-1}$, Extended power rule $(u^n)' = nu^{n-1} \cdot u'$

Chain rule $f(u)' = f'(u) \cdot u'$ or df/dt =(df/du)(du/dt)

Exponentials: $(e^x)' = e^x$, $(e^u)' = e^u u'$; $(b^x)' = b^x \cdot \ln(b)$, $(b^u)' = b^u \cdot \ln(b) \cdot u'$,

Logarithm: $(\ln x)' = 1/x$ $(\ln u)' = u'/u$

Trig: $(\sin x)' = \cos x$, $(\sin u)' = (\cos u) \cdot u'$, $(\cos x)' = -\sin x$ (radian measure assumed).

Other formulas: Average slope = $\dfrac{f(b)-f(a)}{b-a}$, Instantaneous slope = $f'(a)$

Exponential growth: $A = A_0 e^{kx}$