Math U141 Fall 07 Q3 Prof. A. Iarrobino Name ____Solutions_____

Problems #1,2 will count 5 points each. The third will count 2 pts EC.

1.(5 pts) Find antiderivatives F of the following functions.

A. $f = 3x^2 - 2x + 1$. Find an antiderivative F satisfying F(2)=3.

Sol. F= $x^3 - x^2 + x + C$, $3 = F(2) = 2^3 - 2^2 + 2 + C$, $C = -3$.

Ans. $F = x^3 - x^2 + x - 3$

B. $f = 7 \cdot \sqrt[4]{x^7} - \frac{3}{x^{1.5}} + 4(3x-5)^7 + \frac{3}{x}$. Find a general antiderivative F.

Sol. *Rewrite* $f = 7 \cdot x^{7/4} - 3x^{-1.5} + 4(3x-5)^7 + \frac{3}{x}$

Then. $F = 7 \cdot \frac{x^{7/4+1}}{7/4+1} - 3\frac{x^{-1.5+1}}{-1.5+1} + 4\frac{(3x-5)^8}{8 \cdot 3} + 3\ln|x| + C$

(note division by 3 in third term is because of the chain rule in differentiation: for the same reason $\int \cos(3x) = \frac{\sin(3x)}{3} + C$ *)*

Ans. (*simpler*) $F = \frac{28}{11} \cdot x^{2.75} + 6x^{-1.5} + \frac{(3x-5)^8}{6} + 3\ln|x| + C$

2. (5 pts) A quick frog on Mercury where the acceleration of gravity is -12 feet per $\sec^2$, jumps vertically upward with initial velocity of 16 feet per second, from an 8 foot hill above a pond.

a. Determine its velocity v(t) and position s(t) as a function of time.

Ans. v=-12t+16, $s = -6t^2 + 16t + 8$

b. When is it highest? How high does it leap? **Sol:** *Set v=0.*

Ans. t=4/3 sec., $s(4/3) = \frac{56}{3} = 18\frac{2}{3}$ *feet*

c. When does it land on a lily pad floating on the pond? How fast is it traveling then?

Sol. *Set s=0.* ***Ans.*** $t = \frac{4}{3} + \frac{2}{3}\sqrt{7}$ sec; $v = -8\sqrt{7}$ $ft/\sec$.

d*. Discuss whether the frog gets wet.

Note: *Give an answer and reason using your answer to c.*

3. (2 pts) Six equal garden plots, arranged in 2 rows of three, will enclose a total of 3000 square freet. Outside fencing costs $5 a foot, and inside $2 a foot. Letting x= length of a whole row, and y the length of a whole side, determine x,y so as to minimize the cost. Hint: verify that C=12x+14y.

Sol. *Minimize Cost C= 12x+14y with side condition xy=3000.*

Solve side: $y = 1000/x$ *and substitute in C:* $C = 12x + 14\frac{1000}{y}$. Range x>0.

Find critical point: set C'=0: $C' = 12 + 14(1000)(-x^{-2}) = 0$, $12 - \frac{14(1000)}{x^2} = 0$, $12x^2 = 14(1000)$,

Ans. $x = \sqrt{\frac{14(1000)}{12}} = 10\sqrt{\frac{35}{3}}$. $y = \frac{1000}{x} = \frac{100}{\sqrt{\frac{35}{3}}}$ (*decimal answers OK*, 4 *significant digits.*

Check min: $C'' = 14(1000)(2x^{-3}) > 0$ *for all* $x > 0$, *so C CU : MIN*

Quadratic formula: $ax^2 + bx + c = 0$ has solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

: $\int x^n dx = \frac{x^{n+1}}{n+1}$ +C if n ≠ -1, $\int x^{-1} dx = \ln(x)$ +C, $\int b^x dx = \frac{b^x}{\ln(b)}$ +C. , $\int e^{ax} = \frac{1}{a}e^{ax}$ +C

$\int \sin(ax) = -\frac{1}{a}\cos(ax)$, $\int \cos(ax) = \frac{1}{a}\sin(ax)$