Math U141 Fall 07 Prof. A. Iarrobino Name ________________________

Problems #1,2 will count 5 points each. The third will count 2 pts EC.

1.(5 pts) Find antiderivatives F of the following functions.

A. $f = 3x^2 - 2x + 1$. Find an antiderivative F satisfying F(2)=3.

Ans. $f = x^3 - x^2 + x - 3$.

B. $f = 7 \cdot \sqrt[4]{x^7} - \dfrac{3}{x^{1.5}} + 4(3x-5)^7 + \dfrac{3}{x}$. Find a general antiderivative F.

Ans. $\dfrac{28 \cdot x^{11/4}}{11} + \dfrac{6}{\sqrt{x}} + \dfrac{(3x-5)^8}{6} + 3\ln|x| + C$

2. (5 pts) A quick frog on Mercury where the acceleration of gravity is -12 feet per $\sec^2$, jumps vertically upward with an initial velocity of 18 feet per second, from a 60 foot hill above a pond.

a. Determine its velocity v(t) and position s(t) as a function of time.

b. When is it highest? How high does it leap?

c. When does it land on a lily pad floating on the pond? How fast is it traveling then?

d*. Discuss whether the frog gets wet.

a. **Sol.**

$a = -12$

$v = \int a = \int -12 = -12t + c_v;\ v(0) = 18 = -12(0) + c_v,\ so\ c_v = 18,\ and$

$v = -12t + 18.$

$s = \int v = \int -12t + 18 = -6t^2 + 18t + c_x,\ s(0) = 60 = 0 + c_x;$

$s = -6t^2 + 18t + 60.$

b. **Sol:** *highest: set v=0, -12t+18=0, t=1.5 sec., height s(1.5)=73.5ft.*

c. **Sol.** *Set s=0, -6(t-5)(t+2)=0, t=5, vel v(5)=-42 ft/sec*

3. (2 pts) Six equal garden plots, arranged in 2 rows of three, will enclose a total of 3000 square freet. Outside fencing costs \$5 a foot, and inside \$2 a foot. Letting x= length of a whole row, and y the length of a whole side, determine x,y so as to minimize the cost. Hint: verify that C=12x+14y.

v

Sol. C=outside cost + inside cost = 5(2x+2y)=2(x+2y)=12x+14y. (F *to minimize)*

Side: xy=3000.

Step 2: solve side and substitute in F: $y = 3000/x,\quad C = 12x + 14(3000x^{-1})$. *range: x>0.*

Step 3. Solve max-min problem on the interval: Set C'=0

$C' = 12 + 14(3000)(-x^{-2}) = 0;\quad 12 = \dfrac{14(3000)}{x^2};$

$12x^2 = 14(3000),\quad x = \sqrt{\dfrac{14(3000)}{12}},\ x = \sqrt{3500} = 59.16\ ft.$

y-3000/x=50.7 ft.

check for min cost: $C'' = 14(3000)(-(-2x^{-3}) > 0$ *as* $x > 0$, *so C is CU, and we have global min.*

Quadratic formula: $ax^2 + bx + c = 0$ has solution $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

: $\int x^n dx = \dfrac{x^{n+1}}{n+1}$+C if n ≠ -1, $\int x^{-1} dx = \ln(x)$ +C, $\int b^x dx = \dfrac{b^x}{\ln(b)}$+C. , $\int e^{ax} = \dfrac{1}{a} e^{ax}$ +C

$\int \sin(ax) = -\dfrac{1}{a}\cos(ax),\ \int \cos(ax) = \dfrac{1}{a}\sin(ax)$