Half quiz 3.5B Math U141 Prof. A. Iarrobino **Solutions**

Please show all work, for full credit. The best three of four problems will count four points each..

1. Given that $f(x) = 3x,\ for\ -2 \le x \le 4$ Graph f, and determine $\int_{-2}^{4} f(x)\, dx$ in two ways

i. geometrically as signed area, and **Ans.** i. There is one triangle below the axis, with vertices (-2,0), (-2,-6), and (0,0) of area 6·2/2), and a second triangle above the axis, with vertices (0,0), (4,0), (4,12), of area (4·12/2)=24, so the integral is 24-6=18.

ii. using the fundamental theorem.

Ans. $\int_{-2}^{4} 3x\ dx = \left.\frac{3x^2}{2}\right]_{-2}^{4} = \frac{3(4)^2}{2} - \frac{3(-2)^2}{2} = 24 - 6 = 18.$

2. Consider the following table of water flow over a dam on the Wandering Creek: time is in minutes since 9 AM, and flow rate is in cubic meters/hour

t	0	5	10	15	20
r	50	70	100	90	80

Estimate the total amount of water that has gone over the dam in the first 20 minutes, Including units, using

i. left sum **Ans.** 5(50+70+100+90)=1550 $\frac{m^3}{hr}\cdot \min = \frac{1550}{60} m^3 = 25\frac{5}{6}\ m^3.$

ii. trapezoid sum **Ans.** $5\left(\frac{1}{2}\cdot 50 + 70 + 100 + 90 + \frac{1}{2}\cdot 80\right)\frac{m^3}{60} = \frac{1625\ m^3}{60} = 27.0833\ m^3$

3. The ice on Tadpole Pond is 2" thick, at 9 AM, and the thickness T in inches is changing at the rate of $\frac{\sqrt[3]{x}}{120},\ inches/\min ute,\ x\ \ hours\ \ after\ 9AM.$

a. Write an integral for the change in thickness from 9 AM to x hours after 9 AM.

Ans. $\int_0^x \frac{\sqrt[3]{x}}{120}\ dx\ \frac{inches}{\min}\cdot hrs = \int_0^x \frac{\sqrt[3]{x}}{2} dx \quad inches$

b What is the thickness of the ice at 10 AM? (give units)

Ans. *Change in thickness is* $\int_0^1 \frac{\sqrt[3]{x}}{2} dx = \int_0^1 \frac{x^{1/3}}{2} dx = \left.\frac{x^{4/3}}{2\cdot(4/3)}\right]_0^1 = \frac{3}{8}(1-0) = \frac{3}{8}\ inches.$

New thickness is 2.375 inches.

4. a. Determine the first, fourth and tenth terms in the right sum R(20) for A= $\int_1^3 x^3 dx$.

Ans. $\Delta x = \frac{3-1}{20} = 0.1. \quad R(20) = 0.1\left(1.1^3\right) + \cdots + 0.1\left(1.4^3\right) + \cdots + 0.1(2^3) + \cdots + 0.1\left(3^3\right).$

(*here first, fourth, tenth and last terms are shown).*

b. Give a formula for the difference R(n)-L(n) between right and left sums with n rectangles, for this integral. **Ans.** *For* $\int_a^b f(x)dx, \quad R(n) - L(n) = \Delta x\left(f(b) - f(a)\right).$ *Here this is* $\frac{2}{n}\left(3^3 - 1^3\right) = \frac{52}{n}.$

c Determine A exactly, using the Fundamental Theorem.

Ans. $\int_1^3 x^3 dx = \left.\frac{x^4}{4}\right]_1^3 = \frac{81}{4} - \frac{1}{4} = 20.$