Math U141 F08 Quiz #1A Prof. A. Iarrobino Name_ **Solutions**

Scoring: #1,2 6 pts. Each; one of #3A or #3B 2pts extra credit. 12=100%.

1. Consider the secant line cutting $y=3x^2-2x-3$ at the points P(2,5) and $Q(2+h,3(2+h)^2-2(2+h)-3)$. Here y= height in feet, x = time in minutes.

A. Find the slope m_{PQ} of the line PQ (the answer is in terms of h)..

Sol. $$m_{PQ}=\frac{\left(3(2+h)^2-2(2+h)-3\right)-5}{2+h-2}=\frac{3(4+4h+h^2)-4-2h-8}{h}=\frac{10h+3h^2}{h}=10+3h\ \ ft/\min$$

B. Find the slope of the graph of $y=3x^2-2x-3$ at x=2, by finding $\lim_{h\to 0} m_{PQ}$.

Ans. $m_P=\lim_{h\to 0} 10+3h=10\ \ ft/\min$

h	0.1	-0.01	0.001	$\to 0$
10+3h	10.3	9.97	10.003	$\to 10$

C. Find the equation of the tangent line to the graph at x=2, using #1B.

Ans. y-5=10(x-2) y=10x-15.

2A. Consider the height function of the Centennial Balloon flown by the Northeastern University Aerial Balloon Society. By examining slopes, determine if the function is linear. If not, explain why not.

	P	Q	R	S
time	9 AM	9:30	10:40	11
height (ft)	32	47	82	90

Sol: $m_{PQ}=15\,ft/30\min=0.5\ \ ft/\min;\ \ m_{QR}=35/70=0.5\ \ ft/\min;\ m_{RS}=8/20=0.4\ \ ft/\min.$

Ans. *Not a straight line,as the slopes are not all the same. But PQR is straight.*

2B. Give equations of the line PQR letting x= time in minutes since 9 AM, y=height of balloon in feet.

Ans. y=0.5x+32

2C Write an equation for the line segment RS using units of yards (one yard = 3 feet) for height and hours for time, with x=0 taken as 9 AM.

Sol. $$m_{RS}=\frac{30-27\frac{1}{3}\ yd.}{2-1\frac{2}{3}\ hr.}=\frac{8/3}{1/3}=8\ \ yd/hr.$$ **Eqn. y-30=8(x-2), y=8x+14.**

2D. Give coordinates for the point S that is 2/3 along the line PQ.

Sol. $$S=P+\frac{2}{3}(PQ)=(0,32)+\frac{2}{3}(\Delta x,\Delta y)=(0,32)+\frac{2}{3}(30,15)=(0,32)+(20,10)=(20,42)$$ (in minutes,feet)

3A*. If a biker averages 20 mph over a 30 mile bike ride, which of the following are always true. Explain why or why not.

a. The biker takes 1.5 hours for the ride.

Ans *TRUE: since by definition, average speed* $=\frac{\Delta\ \text{distance}}{\Delta\ \text{time}}=\frac{30}{1.5}mph.$

b. The biker must be going at 20 mph sometime during the ride

Ans. *TRUE*: since, if the biker starts out slower than 20 mph, she/he must go faster sometime later so as to have an average speed at least 20: so must pass through 20 mph; Similarly, if the biker starts out faster than 20 mph, it must sometime go slower, and pass through 20 mph. ....

c. Assume the same biker averages s mph for the first 10 miles; then determine the average speed for the last 20 miles of the trip.

Sol. *The biker takes* $\frac{10\ miles}{s\ mph} = \frac{10}{s} hrs$ *for the first 10 miles, so takes* $1.5 - \frac{10}{s}\ hours$ *for the last 20 miles, hence the average speed for the last 20 miles is* $\frac{20\ m}{\left(1.5 - \frac{10}{s}\right) mph} = \frac{20s}{1.5s - 10}\ hours.$

OR

3B*. Approximate the derivative of 4^x at x=0. Set up the difference quotient, giving the slope m_{PQ} from the point P(0,1) to Q(h, 4^h), then use a small value of h to approximate the slope m_P. (Give the value of h used).

Sol. *The difference quotient is* $\frac{4^h - 1}{h - 0} = \frac{4^h - 1}{h}.$

We have

h	0.01	0.001	0.00001	10^{-11}
$\frac{4^h - 1}{h}$	1.396	1.387	1.386	1.38629

Ans. *The derivative of* 4^x *at x=0 is approximately 1.38629*
(Note: we weill see later that it is actually ln(4))