

Math U141 Quiz 2 F08 Prof. A. Iarrobino **Solutions**  
 Scoring: #1 6 pts, #2 5 pts, #3 5 pts; Scale: 15pts=100%

1. The depth of a pollywog in inches below the surface of Houghton Pond is given by  $d(x) = 36x - 2x^2$ ,  $x$  minutes after the beginning of its dive, until it returns to the surface.

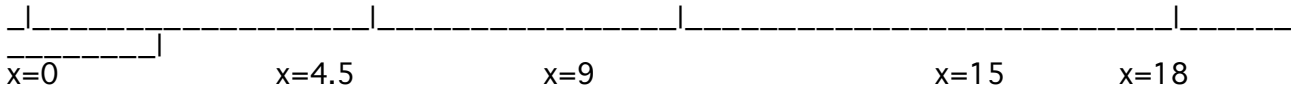
- A. When does the pollywog return to the surface? **Ans.  $x=18$  min**  
 B. What is its average vertical velocity between 30 and 150 seconds? **Ans. 30 in/mim**

C. Determine the slope function  $d'(x)$  using a quick formula.

**Ans.  $d'(x) = 36 - 4x$**

D. Graph the depth  $d(x)$  as a function of time  $x$ . Also graph  $d'(x)$  on the same axes.  
 $y=175$

**Ans in class: a parabola with apex and max at (9, 162), and a line thru (0,36), (9,0)**



E. For what value  $x_1$  of  $x$  is  $d'(x)=0$ ? Sol:  $36-4x=0$ , **Ans.  $x_1 = 9$  in**

What is the pollywog doing at time  $x_1$ ?

**Ans. turning around from descending to ascending**

What special point is  $(x_1, d(x_1))$  on the graph of the function  $d(x)$ ?

**Ans. (9,162) is MAXIMUM of  $d(x)$ , and apex of parabola**

F. a. Find the equation of the tangent line (TL) to  $y=d(x)$  at (15,90).

**Sol: slope TL  $m=d'(15)=36-4(15)=-24$  in/min. Ans  $y-90=-24(x-15)$ ,  $y=-24x+450$**

b. For what value of  $x$  does this TL cross the  $x$ -axis? **Sol:  $y=0$ , Ans.  $x=18.75$  min.**

Explain what significance this has for the pollywog (interpret motion "along" the TL).

*If the pollywog continued from 90 inches deep at 15 minutes, at a constant speed of 24 in/min, it would reach the surface at 18.75 minutes.*

G \*. What is a pollywog? (EC for either accuracy, or interesting answer - not just "tadpole").

2. Find the derivative functions  $f'(x)$  for the following functions  $f(x)$ .

2A.  $f(x) = 3x^5 - 2x^e + 7x^{0.3} - \frac{1}{x^2}$ . **Ans.  $15x^4 - 2ex^{e-1} + 2.1x^{-0.7} + 2x^{-3}$ .**

2B.  $f(x) = \sqrt[5]{x^8} - \frac{3}{\sqrt[4]{x^7}} + 2$  **Ans.  $\frac{8}{5}x^{3/5} + \frac{7}{4}x^{-11/4}$**

2C (chain rule) For  $f(x) = \sqrt{x^3 - 9x}$  please write  $f(x) = g(u)$ : specify the outside function  $g(x)$  and the inside function  $u(x)$ , then find the derivative  $f'(x)$ .

**Ans:  $g(x)=\sqrt{x}$ ,  $u(x)=x^3 - 9x$  Slope  $f'(x) = \frac{1}{2}(x^3 - 9x)^{-1/2}(3x^2 - 9)$**

3A. Assume that the function  $f(x)$  satisfies  $f(3)=19$ ,  $f'(3)=5$ , and  $f''(3)=-4$ . Using approximation along the tangent line, estimate the value of  $f(3.4)$

**Sol: TL:  $y-19=5(x-3)$ ,  $y=5x+4$ ,  $y(3.4)=21$  Ans: 21.**

Also, determine whether your estimate is too high or too low. **Ans. Too high**

**Because  $f$  is concave down at  $P(3,19)$ , so TL is above graph.**

3B. i. When  $f(x)$  is increasing, the derivative  $f'(x)$  is positive.

ii. When the derivative  $f'(x)$  is decreasing the function  $f(x)$  is concave down.

iii. *The derivative function tells us the slope of the original function at each point.*

3C. Answer to graph of derivative given  $f(x)$ : in class.