Math U141 Quiz 3 Fall 08 Prof. A. Iarrobino Solutions_______________

Tony Iarrobino 11/13/08 7:33 PM
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Problems #1.2.3 will count 6 points each.
Please show all work for full credit. Scale: 17=100%

1.(5 pts) Find antiderivatives F of the following functions.
A. $f = 3x^2 - 8x + 7$. Find an antiderivative F satisfying F(1)=21.

Sol. $F = x^3 - 4x^2 + 7x + C \quad F(1) = 21, 4 + C = 21,\ C = 17.$ **Ans.** $F = x^3 - 4x^2 + 7x + 17$

B. $f = 8 \cdot \sqrt[4]{x^5} - \frac{2}{x^{3.5}} + \sin(3x) + e^{2x} + 1$. Find a general antiderivative F.

Ans. $F = \frac{32}{9}x^{2.25} + 0.8x^{2.5} - \frac{\cos(3x)}{3} + \frac{e^{2x}}{2} + x + C$

2. (6 points). An urban school wishes to build three rectangular garden plots in a row, with a total area of 1050 square feet. If exterior fence costs $6 per linear foot, and interior $1 per foot, and the outside dimensions of the plot are x (row length), and y, determine x,y so as to minimize the cost of the fence. *Hint: Show that cost C=12x+14y.*

Sol. *Minimize C=12x+14y. Side condition xy=1050;* $y = 1050/x$ *x>0.*

Sub: $C = 12x + 14\frac{1050}{x} = 12x + 14(1050)x^{-1}.$

$C' = 12 + 14(1050)(-x^{-2}) = 0,\ \ 12 = \frac{14(1050)}{x^2},\ x^2 = \frac{14(1050)}{12} = 7^2 \cdot 5^2$ *x=35, y=30.*

Check min: $C'' = (14)(1050)(2x^{-3}). > 0,$ *C Concave up for* $x > 0$*, so MIN. C(0)=∞, C(35)=840.*

2. (6 pts) A quick young frog on Politis (an imaginary planet) where the acceleration of gravity is -0.4 feet per $\sec^2$, jumps vertically upward with an initial velocity of 2 feet per second, from the top of an 8 foot hill above a pond.

a. Determine its velocity v(t) and position s(t) as a function of time. Please include units in your answer.

Sol $v=\int a\ dt=\int -0.4\ dt=-0.4t+c_v \quad v(0)=2=-0,4(0)+c_v,\ c_v=2.$

Ans. v(t)=-0.4t+2

Sol. $_s=\int v\ dt=\int(-0.4t+2)\ dt=-0.2t^2+2t+c_s \quad s(0)=8=-0,2(0^2)+2(0)+c_s,\ c_s=8.$__________

Ans. s(t)= $-0.2t^2+2t+8$

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b. When is the frog highest? How high is it above the pond then?

Sol. *Set v(t)=0 -0.4t+2=0, so t=5.*

Ans. t= 5 sec., s(5)=13 ft.________________

c. When does the frog pass an osprey on a platform ~~14.4~~ 12 feet above the pond?

Sol. *Set s(t)=* $-0.2t^2+2t+8=12$*,* , $-0.2t^2+2t-4=0,\ t=\dfrac{-2\pm\sqrt{2^2-4(-0.2)(-4)}}{2(-0.2)}=5\pm\sqrt{5}$

Ans. t= _2.7634 secs (on way up)_and t= _7.236 secs (down)________.

d. The frog lands on a lily pad floating on the pond. When does it land? What is its velocity as it lands (not zero)?

Sol Set $-0.2t^2+2t+8=0$. *solve by quad eqn, substitute answer into v=-0.2t+2.*

Ans. t= _13.0623 secs___ v= _-3.22 ft/sec___

4* EC (2 pts) Group fare for a bus excursion to Montreal is $80 each person if 20 persons go, and the price decreases by 0.50/person for each additional 2 people. What group size maximizes the total amount collected from the group?

Sol. *Let y = price, x = # persons in group (question, "what is group size").*

Then m $=\dfrac{\Delta y}{\Delta x}=\dfrac{-\$0.50}{2\ \textit{persons}}=-0.25\ \ \$/\textit{person}.$ *y-80=-0.25(x-20), y=-0.25 x+85. Range: 0≤x≤340.*

Maximize Total collected T= xy=$-0.25x^2+85x$ *Set T'=0, -0.25(2x)+85=0, x= 170 persons.*
Check Max: SDT T"=-0.5 always negative, so T CD, and x=170 gives a max.
Or form table using (x=0, T=0); (x=170, T=170·(42.5) (340,0)., shows that x=170 is MAX.

Quadratic formula: $ax^2+bx+c=0$ has solution $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

: $\int x^n dx=\dfrac{x^{n+1}}{n+1}$+C if n ≠ -1, $\int x^{-1}dx=\ln(x)$ +C, $\int b^x dx=\dfrac{b^x}{\ln(b)}$+C. , $\int e^{ax}=\dfrac{1}{a}e^{ax}$ +C

$\int\sin(ax)=-\dfrac{1}{a}\cos(ax),\ \int\cos(ax)=\dfrac{1}{a}\sin(ax)$