Math U142 Spr06 Prof. A. Iarrobino Quiz 1. Name_____

Some integral and other formulas appear at end. Good luck!

1.

A. Determine y'=dy/dx if $x^3 - 6y^2 = 10.$

B. A pulley is on the edge of a dock, 9 feet above the water level. A rope Is being used to pull in a boat. The rope is attached to the boat at water level, and is being pulled in at a rate of 2 ft per second. Find the rate at which the boat is approaching the dock at the instant the boat is 12 feet from the dock.

C. A sphere has a measured radius of 3.03 inches, with a possible error of 0.002 inches. Estimate the maximum error in the volume of the sphere.

2 . Find the following integrals using substitution and standard formulas

A. $\int \left(x^5 + 5x\right)^{-2} (x^4 + 1)dx, \;\; let\; u = x^5 + 5x$

B. $\int_0^{\pi/4} \left(\sin x\right)^3 \cos x \, dx, \; let\; u = \sin x.$ (Give related u,du integral with new limits)

C. $\int_0^1 \frac{x+3}{\sqrt{x+2}} dx, \; u = x + 2.$ (Give related u,du integral with new limits)

2. Find the following integrals using the tables at end.

A. $\int \frac{dx}{x^2+13}$

B. $\int \sqrt{(5x-3)^2+16}\, dx$ *Hint : first let* $u = 5x - 3.$

C. $\int \frac{3e^x dx}{1+e^{2x}}.$ *substitute* $u = e^x.$

3. Integral and meaning of integral

3A. The following table gives the rate of flow of water into a reservoir, t minutes after a valve is opened, in cubic feet/minute.

Estimate the total amount of water that has flowed into the reservoir in the first 6 minutes, using a. right sum b. left sum c. trapezoid sum (average of left and right sums).

t min:	0	1.5	3	4.5	6
flow	6	10	11	7	4

On a graph that includes the points specified above, indicate which rectangles comprise the left sum.

3B. A function f defined on the interval [0,4] satisfies, f(0)=0, f(2)=6, and f(4)=0, and consists of two line segments.

a. Determine $\int_0^4 f(x)dx$, using the meaning of the integral and geometry.

b. Determine the two lines comprising f(x).

3C. The following graph shows the velocity of a bicyclist. Determine

a. The total distance the cyclist travels between 9 AM and 9:30 AM.

b. The time of maximum acceleration of the bicycle, and an estimate of the maximum acceleration.

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Formulas:

Antiderivatives: a. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ if n ≠ -1, b. $\int x^{-1} dx = \ln(x) + C$,

c. $\int b^x dx = \frac{b^x}{\ln(b)} + C$. d. $\int \sin(ax) \cdot dx = \frac{-1}{a}\cos(ax)$, e. $\int \cos(ax) \cdot dx = \frac{1}{a}\sin(ax)$,

f. $\int e^{ax} dx = \frac{e^{ax}}{a}$ g. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right|$

i. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ j. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$

Other:

Area of triangle A=bh, Area of trapezoid A=$(b(h_1 + h_2)/2$.

Area of circle: πr^2. Area of ellipse: πab where a, b are the short and long radii.

Volume of sphere: V=$\frac{4}{3}\pi r^3$