Math U142 Calculus 2 Spring 04 Prof. A. Iarrobino   WS #2

Determine the volumes of the following solids of revolution, swept out by rotating a given region about the specified axis.

1A. A horn with square cross-section is constructed so that the bottom sides of the squares lie on the x-y plane, between y=x^3 and y=0, from x=2 to x=5. Determine its volume.
1B. A similar horn is constructed so that the bottom sides lie on the x-y plane, between y=x^3 and y=25x, from x=2 to x=5. Determine its volume.

2A. The bounded region between $y = x^3,\ and\ \ y = 4x$ (in the first quadrant) is rotated about the x-axis. Set up an integral for the volume swept out, using the washer (``ring`` or ``annulus``) method. Then evaluate the integrals to find the volume.

2B. How is your integral changed, if the bounded region in #2A is rotated instead about y=10?

3A. The bounded region between $y = x^2 - 30,\ and\ \ y = 10 + 6x$ is rotated about the axis y=-30. Describe the resulting shape, and determine its volume.

3B. The same bounded region in #3A is rotated about the axis y=-14. What issue of overlap arises in determining the volume swept out? Find the volume.

4. The region between y=4x, and y=x^3, from x=1 to x=3, is rotated about the x-axis. Determine the volume swept out. *Hint: Determine where the two curves cross, and use two integrals).*

Formulas:

Cross-section or “slice of bread” method:

V=$\int A(x)dx$ (here A(x) is the area of a cross section perpendicular to x-axis)

Disk method:

V=$\int \pi r^2 dx$ (volume of revolution about axis y=a parallel to x-axis; here r=y-a).

:V=$\int_a^b \pi f(x)^2 dx$ (region between y=f(x) and x-axis, x=a,x=b is rotated about x-axis.

Washer method:

V=$\int_a^b \pi\left(R^2 - r^2\right)dx$ (region bounded by y=f(x), y=g(x) is rotated about axis y=a:

here R= big radius, r = small radius).